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Soft Bornological Structures

A Thesis

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by

Ashwaq Fawze Abdal

Supervisor

Assist. Prof. Dr. Anwar Nooruldeen Imran

Prof. Dr. Lieth Abdul lateef Majed

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Signature:
Name: Anwar Nooruldeen Imran
Scientific rank and certificate: Assist. Prof. Dr.
Date: / / 2022

Signature:
Name: Prof. Dr. Lieth Abdul lateef Majed
Scientific rank and certificate: Prof. Dr.
Date: / / 2022

Signature:

Name: Assist. Prof. Dr. Rokan Khaji Mohammed

Date: / / 2022

(Head of Mathematics Department)

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Signature:

Name: Amal Shaker Hameed
Title: Lecturer Doctor
Date: / / 2022

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Signature:

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Title: Professor Doctor
Date: / / 2022

Signature:

Name: Alaa Mahmood Farhan
Title: Assistant Professor Doctor
Date: / / 2022

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(Member) Signature:
Name: Assist. Prof. Dr. Rabeaa Gatea Abtan
Date: / / 2022

(Member) Signature:

Name: Lecturer Dr. Ghazwan Mohammed Jaafar

Date: / / 2022

(Member and Supervisor) Signature:

Name: Assist. Prof. Dr. Anwar Nooruldeen Imran

Date: / / 2022

(Member and Supervisor) Signature:
Name: Prof. Dr. Lieth Abdul lateef Majed

Date: / / 2022

Approved by the Dean of College of Science, University of Diyala

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Signature:
Name: Prof. Dr. Tahseen H. Mubarak
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الاهداء

الفضل ينُسب لله رب العالمين أولاً، وهو من جعل الفضل مُتداوَلاً بين عباده، ولن ينُزع إلى قيام الساعة.

إلى من شرفني بحمل اسمه وبذل الغالي والنفيس في سبيل وصولي لدرجة علمية عالية

والدي العزيز حفظه الله تعالى ...

إلى نور عيني وضوء دربي ومهجة حياتي التي دعواتها وكلماتها رفيق الألق والتفوق

امي ثم امي ثم امي ...

إلى أخوتي وأخواتي اصحاب المكانة الغالية في قلبي

الى اخي الشهيد البطل أثير فوزي عبدال رحمه الله...

الى زميلاتي ورفيقات الدرب اللواتي لم يبخلن عليَّ بالمساعدة

إلى كل من وقف بجواري أهُديكم جميعاً رسالتي هذه...

دمتم في حفظ الله ورعايته

الشكر والتقدير

ليس بعد تمام العمل من شيء أجمل ولا احلى من الحمد، فالحمد لله والشكر له كما ينبغي لجلال وجهه وعظيم سلطانه وكما ينبغي لجزيل فضله وعظيم احسانه على ما انعم به علي من اتمام هذا البحث المتواضع.

ثم انه لا يسعني ألا ان اشيد بالفضل واقر بالمعروف لكل من ساهم في انجاز هذا البحث واخص بالذكر اعترافا بالجميل.

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LIST OF ABBREVIATIONS

B	Bounded Set
X	Nonempty Set
β	Bornology
(X,β)	Bornological Set
β_0	Base of Bornology
(G,β)	Bornological Group
$\mathbb{X}$	Nonempty Soft Set
$\mathbb{G}$	Soft Group
$\tilde{B}$	Soft Bounded Set
$\tilde{\beta}$	Soft Bornology
$(\mathbb{X},\tilde{\beta})$	Soft Bornological Set
$\tilde{\beta}_0$	Soft Base of Soft Bornology
$\tilde{\Gamma}$	A family of all soft unbounded subsets of $\mathbb{X}$
$(\mathbb{G},\tilde{\beta})$	Soft Bornological Group
$P(X)$	Power Set of X
S_n	Symmetric Group
$\leq, <$	Subgroup, Proper Subgroup
$\lhd$	Normal subgroup
$\tilde{\subseteq}$	Soft Subset
$\tilde{\cup}, \tilde{\cap}$	Union of Soft Set, Intersection of Soft Set
$\tilde{+}$	Soft Addition
$\preccurlyeq$	Partial Ordered Relation

$\tilde{\setminus}$	Soft Difference
$\mathbb{G}/_{\mathbb{H}}$	Quotient Soft Bornological Group
θ_e	Soft Bounded Action Map
$\mathbb{G}(x)$	The Soft Orbit of x Under The Action $\mathbb{G}$

ABSTRACT

A " bornology " is a structure to solve the problems of boundedness for sets, groups, and functions in a general way. The main goal of this work is to combine the soft set theory with bornology to construct a new structure which is called a soft bornology to solve the problems of boundedness for the soft sets and soft groups. Also, we construct the soft base and soft subbase for this structure. It is natural to study fundamental constructions for this new structure such as soft bornological subset and product soft bornology. As well as, a new structure whose elements are soft unbounded sets are generating. Finally, we study a soft bounded action that means, when a soft bornological group acts on a soft bornological set, this process is called a soft bornological group action or soft bounded action. The effect of the soft bounded action is to partition a soft bornological set into classes of soft orbitals. The main important results, we will prove that a family of soft bornological sets can be a partial ordered set by a partial ordered relation, the composite of two soft bounded maps is a soft bounded map, the intersection of soft bornological sets is a soft bornological set, the left- right translation is a soft bornological isomorphism, the product of soft bornological groups is also a soft bornological group and the soft bornological group action is a soft bornological isomorphism.

INTRODUCTION

There is a very important reason to construct the structure of bornology. In the beginning when researchers wanted to solve the problems of boundedness they gave the concept of a bounded set on the real numbers $\mathbb{R}$, also in a topological vector space and a metric space the set is bounded if it is absorbent by every neighborhood of zero, as a ball respectively. That motivated the researchers to construct such kind of structure in functional analysis, which have the minimum amount of conditions to solve the problem of boundedness for any set in a general way, which is called bornological structures. H. Hogbe–Nlend [5], [6] gave a general and integrated definition that solves the problem of boundedness. After that, Pombo [18] gave a new structure called bornological group to solve the problem of boundedness for the groups and studied the fundamental construction for this structure. Bambozzi [3] studied the category theory of bornological group. Anwar N. Imran [8] studied bornological semigroup to solve the problem of boundedness for such kind of groups which cannot be bornological group because the inverse map is not bounded. After that, Anwar N. Imran [9] added additional properties of the bornological group, also Anwar N. Imran [11] formed a new structure to solve the problem of boundedness to a type of groups whose product map is not bounded. As well as, Anwar N. Imran [10] started to solve many existent problem in algebraic bornology and many researchers studied bornology like [2], [21].

In 1999, Molodtsov [15] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties in modelling problems in engineering physics, economics, computer science, social sciences and medical sciences [13], [14] and [17]. Many researchers have contributed to develop soft set theory's algebraic structure to the best [1], [4], [7] and [16].

The main goal of this study is to combine soft set theory with bornology to construct new structures called soft bornological structures.

In chapter one, we recall some basic concepts for bornological structures to solve the problem of boundedness for sets and groups. Also, the fundamental construction for its. The most important part in this chapter is the practical applications of bornological structure such as in the spyware program KPJ, in the case of need to determine a person’s location or the identity of a person from his finger print or from an eye print, and in satellite broadcast systems to determine the limits of the broadcast area [2].

In chapter two, we combine the soft set theory with bornology to construct a new structure that is called a soft bornology to solve the problems of boundedness for soft sets. Also, we construct a soft base and a soft subbase for this structure. It is a natural to study fundamental construction for this new structure such as soft bornological subset, product soft bornology. Consequently, we study soft bornological group to solve the problems of boundedness for soft group. The main important results, we prove that a family of soft bornological sets can be partial ordered set, every soft power set of a soft set is soft bornological set, the composition of two soft bounded maps is a soft bounded map, the intersection of soft bornological sets is a soft bornological set, the left-right translation is soft bornological isomorphism and the product of soft bornological groups is a soft bornological group.

In chapter three, we study soft bornological group actions denoted by SBGA, when a soft bornological group $\mathbb{G}$ acts on a soft bornological set $\mathbb{X}$. The soft bornological set $\mathbb{X}$ is called a $\mathbb{G}$- soft bornological set. The effect of the soft bounded action is to partition a soft bornological set into classes of soft orbitals. The main important result, we prove that soft bornological group action is soft bornological isomorphism.

Chapter One

Some Basic Concepts for Bornological Structures

1.1 Introduction

In this chapter, we recall some basic concepts for bornological structures to solve the problem of boundedness for a set and a group. It is a natural to study fundamental construction for this structure such as bornological subset, product bornological set. Also, the most important part in this chapter is the practical applications of bornological structure; see [2].

1.2 Bornological Set

In this section, we recall some basic concepts for the bornological set with some examples and fundamental constructions for this structure.

Definition 1-2-1 [5]:

Let X be a nonempty set. A ***bornology*** β on X is a collection of subsets of X such that:

i. β forms a cover of X;

ii. β inclusion under hereditary, i.e. if $B \in \beta, \exists\, A \subseteq B$ then $A \in \beta$;

iii. β inclusion under finite union, i.e. if $\forall\, B_1, B_2 \in \beta$ then $B_1 \cup B_2 \in \beta$.

A pair (X, β) consisting of a set X and a bornology β on X is called ***a bornological set***, and the elements of β are called bounded subsets of X.

We can satisfy the first condition in different ways, if the whole set X belong to the bornology or $\forall\, x \in X, \{x\} \in \beta$ or $X = \bigcup_{B \in \beta} B$.

Definition 1-2-2 [5]:

Let (X, β) be a bornological set. A ***base*** β_0 is a sub collection of bornology β, and each element of the bornology is contained in an element of the base.

Example 1-2-3:

Let $X = \{1,3,5\}$, $\beta = \{\emptyset, X, \{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}\}$.

To satisfy that β is a bornology on X.

i. Since $X \in \beta$, then X is covering itself.

ii. If $B \in \beta, A \subseteq B$, then $A \in \beta$

Since β is the set of all subsets of X, i.e. $\beta = P(X) = 2^X$.

Then β inclusion under hereditary.

iii. β inclusion under finite union, i.e. $\bigcup_{i=1}^{n} B_i \in \beta, \forall B_1, B_2, \dots, B_n \in \beta$.

Since

$\{1\} \cup \{3\} = \{1,3\}$, $\{1,3\} \cup \{1,5\} = X$, $\emptyset \cup \{1,3\} = \{1,3\}$,

$\{1\} \cup \{5\} = \{1,5\}$, $\{1,3\} \cup \{3,5\} = X$, $\emptyset \cup \{3,5\} = \{3,5\}$,

$\{3\} \cup \{5\} = \{3,5\}$, $\{1,5\} \cup \{3,5\} = X$, $\emptyset \cup \{1\} = \{1\}$.

Then β is bornology on X. We can define only one bornology on finite set it is called discrete bornology.

Now, to find the base for bornology

$\beta_0 = \{\{1,3\}, \{1,5\}, \{3,5\}, X\}$, or $\beta_0 = \{X\}$.

Example 1-2-4 [12]:

Let $\mathbb{R}$ be the set of real numbers (with Euclidean norm (absolute value)).
We want to define a usual bornology on $\mathbb{R}$ that mean β is the collection of all usual bounded sets on $\mathbb{R}$, in fact, (a subset B of $\mathbb{R}$ is bounded if and only if there exist bounded interval such that $B \subseteq (a, b)$).

So, $\beta = \{B : B \subseteq (a, b) : \forall a, b \in \mathbb{R}\}$.

i. Since $\forall\, x \in (a, b)$ and $\{x\} \subset (a, b)$ (the bounded intervals with respect to absolute value (w.r.t | |) where the absolute value divided $\mathbb{R}$ into bounded interval), (every subset of bounded interval is bounded). Implies that, $\forall\, x \in \mathbb{R}, \{x\} \in \beta$. Then β covers $\mathbb{R}$.

ii. If $B \in \beta$ and $A \subseteq B$, there is bounded interval such that $A \subseteq B \subseteq (a, b)$.

Therefore $A \in \beta$, and β stable under hereditary.

iii. If $B_1, B_2, \dots, B_n \in \beta$, then there is $L_1, L_2, \dots, L_n$ least upper bound and $g_1, g_2, \dots, g_n$ greater lower bound, such that

$\bigcup_{i=1}^{n} B_i$ has finite union of upper and lower bounds.

Assume that $L = \max\{L_i\}$, and $g = \min\{g_i\}$

$\bigcup_{i=1}^{n} B_i$ has least upper bound L and greater lower bound g.

That mean $\bigcup_{i=1}^{n} B_i$ is bounded subset of $\mathbb{R}$.

That mean $\bigcup_{i=1}^{n} B_i \in \beta$. Then β is bornology on $\mathbb{R}$.

And the base of this bornology is:

$\beta_0 = \{B_r(x): r \in \mathbb{R}, x \in \mathbb{R}\} = \{(x - r, x + r): r \in \mathbb{R}, x \in \mathbb{R}\}$.

It is clear that every element of that bornology β is contained in an element of the base β_0.

Example 1-2-5:

Let $X = \mathbb{R}^2$ with Euclidean norm

$$\|x\| = \left(\sum_{i=1}^{2} |x_i|^2\right)^{1/2}$$

where $x = (x_1, x_2)$.

$$D_r(b) = \{x \in \mathbb{R}^2: \|x - b\| \le r, for\ r > 0\}$$

be a disk of the radius r with center at $b = (b_1, b_2)$.

A subset B of $\mathbb{R}^2$ is bounded if there exists a disk with center x_0 $D_r(x_0)$ such that $B \subseteq D_r(x_0)$, $x_0 \in B, r \geq 0$.

Let β_u be a family of all bounded subset of $\mathbb{R}^2$.

$\beta_u = \{B \subseteq \mathbb{R}^2 : B \text{ is usual bounded subset } of\ \mathbb{R}^2\}$.

Then $(\mathbb{R}^2, \beta_u)$ is a bornological set and β_u is called usual bornology on $\mathbb{R}^2$.

i. It is clear that every close disk in $\mathbb{R}^2$ is bounded set that means, $B \in \beta_u$ since $\mathbb{R}^2$ is covered by the family of all disks. Then β_u covers $\mathbb{R}^2$.
ii. If $B \in \beta_u$ and $A \subseteq B$, then there is close disk such that $A \subseteq B \subseteq D_r$. Therefore $A \in \beta_u$.
iii. If $B_1, B_2 \in \beta_u$ then D_{r_1}, D_{r_2} are close disks such that $B_1 \in D_{r_1}, B_2 \in D_{r_2}$.

And $r = Max_{i=1,2}\{r_i\}$. Then $B_1 \cup B_2 \subseteq D_{r_2}$.

Thus, the finite union subsets of $\mathbb{R}^2$. Then β_u is a bornology on $\mathbb{R}^2$.

A base of bornology on $\mathbb{R}^2$ is any sub family such that $\beta_0 = \{D_r(x_0) : r > 0\}$.

Definition 1-2-6 [12]:

Let (X, β) be a bornological set, a family $\mathbb{S}$ of members of a bornology β is said to be ***subbase*** for β if the family of all finite unions of members of $\mathbb{S}$ is a base for β.

Example 1-2-7:

Let $X = \{1, 2, 3\}, \beta = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

Let $S = \{\{1\}, \{2\}, \{3\}\}$.

Then S is subbase for β where $\beta_0 = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, X\}$, or $\beta_0 = \{X\}$ are bases for β.

Remarks 1-2-8:

1. Not every family of subsets of a set X will form a base for a bornology on X.

For example $X = \{1,3,5\}$, $\beta_0 = \{\{1\},\{3\},\{5\}\}$ then β_0 cannot be a base for any bornology because the base must be contain a large subsets of X see [12].

2. Let $(X, \beta), (Y, \beta')$ be bornological sets and $\beta_0, {\beta_0}'$ are bases for β and β', respectively. If $\beta_0 \subseteq {\beta_0}'$ then, $\beta \subseteq \beta'$.

For example: $X = \{1, 2, 3\}, Y = \{1, 2, 3, 4\}$

$\beta = \big\{\emptyset, X, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1,3\}, \{2, 3\}\big\}$,

$\beta' = \{\emptyset, Y, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\},$

$\{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$.

$\beta_0 = \{\{1, 2\}, \{1,3\}, \{2, 3\}, X\}$,

${\beta_0}' = \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, Y, \{1,2\}, \{1,3\}, \{2,3\}, X\}$.

It is clear that $\beta_0 \subseteq {\beta_0}'$ then, $\beta \subseteq \beta'$.

3. Types of a bornology:
   a. The discrete bornology β_{dis} is the collection of all subsets of X, i.e. discrete bornology $P(X) = 2^X$.
   b. The usual bornology β_u is the collection of all usual bounded subsets of X, i.e. usual bornology $= \{B \subseteq X : B\ is \text{ usual bounded}\}$.
   c. The finite bornology β_{fin} is the collection of all finite bounded subsets of X.

Definition 1-2-9 [5]:

Let (X,β) and (Y,β') be bornological sets. A map $\psi:(X,\beta)\to(Y,\beta')$ is called ***bounded map*** if the image for every bounded set in (X,β) is bounded set in (Y,β'). That means, $\forall B\in\beta\Rightarrow\psi(B)\in\beta'$.

Notice that, the composition of two bounded maps is bounded map.

Definition 1-2-10 [12]:

A map ψ between two bornological sets (X,β) and (Y,β') is called a ***bornological isomorphism*** if ψ is bijective also, $\psi\ ,\psi^{-1}$ are bounded maps.

Consequently, for every $B\in\beta$ there is $B'\in\beta'$, such that

$$B'=\psi(B),\,B=\psi^{-1}(B').$$

Example 1-2-11:

Consider $X=\{10,11\},Y=\{5,6\}$ with finite bornology on X,Y.

$$\beta=\big\{\emptyset,X,\{10\},\{11\}\big\},\beta'=\big\{\emptyset,Y,\{5\},\{6\}\big\}.$$

Define a function $\psi:X\longrightarrow Y$ such as, $\psi(10)=5,\psi(11)=6$.

It is clear that ψ is bijective and ψ and ψ^{-1} bounded map.

$$\psi(\{10\})=\{5\}\ \Rightarrow\psi^{-1}(\{5\})=\{10\}$$

$$\psi(\{11\})=\{6\}\Rightarrow\psi^{-1}(\{6\})=\{11\}$$

$$\psi^{-1}(Y)=X,\psi^{-1}(\emptyset)=\emptyset.$$

Proposition 1-2-12:

If $\psi:(X,\beta)\longrightarrow(Y,\beta')$ and $\phi:(Y,\beta')\longrightarrow(Z,\beta'')$ are both bornological isomorphism, then the composition $\phi\circ\psi:(X,\beta)\longrightarrow(Z,\beta'')$ is bornological isomorphism.

Proof:

Since ψ and ϕ are one to one and onto, then $\phi \circ \psi$ is one to one and onto (composition of any two one to one, onto maps is one to one, onto map respectively). Since ψ and ϕ bounded maps, then $\phi \circ \psi$ is bounded map.

Since ψ and ϕ are bornological isomorphism, then ψ^{-1} and ϕ^{-1} are bounded also $\psi^{-1} \circ \phi^{-1}$ is bounded, but $\psi^{-1} \circ \phi^{-1} = (\phi \circ \psi)^{-1}$ is bounded.

Therefore, $\phi \circ \psi$ is bornological isomorphism.

Definition 1-2-13 [5]:

Let (X, β) be a bornological set and let $Y \subseteq X$. Then the collection $\beta_Y = \{B \cap Y:$

$B \in \beta\}$ is a bornology on Y. The bornological set (Y, β_Y) is called bornological subset of a bornological set (X, β) and β_Y is called ***relative bornology*** on Y.

Example 1-2-14:

$X = \{2,4,6\}$, with discrete bornology on X.

$\beta = \{\emptyset, X, \{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}\}$.

And the base of bornology is β_0= $\{\{2,4\}, \{2,6\}, \{4,6\}, X\}$, and let $Y = \{2,4\}$.

Then we have

$\beta_Y = \{\emptyset \cap Y, X \cap Y, \{2\} \cap Y, \{4\} \cap Y, \{6\} \cap Y, \{2,4\} \cap Y, \{2,6\} \cap Y, \{4,6\} \cap Y\}$

$\beta_Y = \{\emptyset, Y, \{2\}, \{4\}\}$. Then the subset of bornology is β_Y.

Definition 1-2-15 [10]:

Let (X, β), and (Y, β') be two bornological sets, $\beta_0 = \{$The family of all $B \times B'$ where $B \in \beta, B' \in \beta'\}$.We say that β_0 is a base, and is called ***product bornology***.

If we defines a bornological structure on $X \times Y$, then the product set $X \times Y$ with this bornological structure is called a bornological product sets of (X, β), and (Y, β').

Example 1-2-16:

Suppose $X = \{a, b\}, Y = \{1, 2\}$.

We defined a discrete bornology on X, Y.

$$\beta = \{\emptyset, X, \{a\}, \{b\}\}$$

$$\beta' = \{\emptyset, Y, \{1\}, \{2\}\}.$$

$$\beta \times \beta' = \big\{\emptyset, X \times \emptyset, X \times Y, X \times \{1\}, X \times \{2\}, \{a\} \times \emptyset, \{a\} \times Y, \{a\} \times \{1\}, \{a\} \times \{2\}, \{b\} \times \emptyset, \{b\} \times Y, \{b\} \times \{1\}, \{b\} \times \{2\}\big\}$$

$$\beta \times \beta' = \big\{\emptyset, X \times Y, X \times \{1\}, X \times \{2\}, \{a\} \times Y, \{a\} \times \{1\}, \{a\} \times \{2\}, \{b\} \times Y, \{b\} \times \{1\}, \{b\} \times \{2\}\big\}$$

$\beta \times \beta' = \{\emptyset, X \times Y, \{(a, 1), (b, 1)\}, \{(a, 2), (b, 2)\}, \{(a, 1), (a, 2)\}, \{(a, 1)\},$

$\{(a, 2)\}, \{(b, 1), (b, 2)\}, \{(b, 1)\}, \{(b, 2)\}\}$

$\beta_0 = \beta \times \beta'$ is a base, and is called product bornology. And if

$X \times Y = \{(x, y): x \in X, y \in Y\}$

$= \{(a, 1), (a, 2), (b, 1), (b, 2)\}$, define a discrete bornology on $X \times Y$.

$\beta_{X \times Y} = \{\emptyset, X \times Y, \{(a, 1)\}, \{(a, 2)\}, \{(b, 1)\}, \{(b, 2)\}, \{(a, 1), (b, 1)\},$

$\{(a, 1), (b, 2)\}, \{(a, 2), (b, 1)\}, \{(a, 2), (b, 2)\}, \{(a, 1), (a, 2)\}, \{(b, 1), (b, 2)\},$

$\{(a, 1), (a, 2), (b, 1)\}, \{(a, 1), (a, 2), (b, 2)\}, \{(a, 1), (b, 1), (b, 2)\},$

$\{(a, 2), (b, 1), (b, 2)\}\}$. $(X \times Y, \beta_{X \times Y})$ is called a bornological product sets.

1.3 Bornological Group

In this section, we recall concept of bornological group to solve the problems of boundedness for group.

Definition 1-3-1 [18]:

A bornological group (G, β) is a set with two structures:

i. $(G,*)$ is a group;
ii. β is a bornology on G.

Such that the product map $\psi: (G, \beta) \times (G, \beta) \longrightarrow (G, \beta)$ is bounded and $\psi^{-1}: (G, \beta) \longrightarrow (G, \beta)$ is bounded.

In the other words, a bornological group is a group G together with a bornology on G such that the group binary operation and the group inverse maps are bounded with respect to the bornology.

Let G be a bornological group and B_1, B_2 be two bounded subsets of G.

We denote for the image of $B_1 \times B_2$ under product map

$$(G, \beta) \times (G, \beta) \longrightarrow (G, \beta)$$

by $B_1 * B_2 = \{b_1 * b_2 : b_1 \in B_1, b_2 \in B_2\}$, $*$ is the binary operation defined on the group which we want to bornologies it, and $B^{-1} = \{b^{-1} \in B\}$.

Example 1-3-2:

Let $G = (\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}_{2\times 2}, \ \cdot\ )$ be a group $a, b \in \mathbb{R}, a \neq 0$, which is the set of matrix elements and multiplication operation.

We can define a finite bornology β on this group, which it is a collection of all finite subsets of G.

To prove G with finite bornology β is bornological group (G, β).

We must prove that, the product map and the inverse map are bounded.

i. $\psi: (G,\beta) \times (G,\beta) \longrightarrow (G,\beta)$.

Let $M_1, M_2 \in (G,\beta)$ be two bounded subsets, we must prove that

$\psi(M_1 \times M_2)$ is bounded.

$\psi(M_1 \times M_2) = M_1 \cdot M_2 = \{m_1 \cdot m_2 : m_1 \in G, m_2 \in G\}$

$$=\{m_1 \cdot m_2 :\ m_1 = \begin{bmatrix} a_1 & b_1 \\ 0 & 1 \end{bmatrix}, m_2 = \begin{bmatrix} a_2 & b_2 \\ 0 & 1 \end{bmatrix}, a_1, a_2, b_1, b_2 \in \mathbb{R}\}$$

$$=\{\begin{bmatrix} a_1 & b_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_2 & b_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_1 \cdot a_2 & a_1 b_2 + b_1 \\ 0 & 1 \end{bmatrix}\} \subset (G,\beta)$$

the product map is bounded.

ii. $\psi^{-1}: (G,\beta) \longrightarrow (G,\beta)$.

Let $M \in (G,\beta), M = \{m : m = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}\}$ which is finite set.

Then $M^{-1} = \{m^{-1} : m^{-1} = \begin{bmatrix} \frac{1}{a} & \frac{-b}{a} \\ 0 & 1 \end{bmatrix}\} \subset (G,\beta)$.

So, ψ^{-1} is bounded. Then (G,β) is bornological group.

Example 1-3-3:

Let $(\mathbb{Z}, +)$ be an additive group and β be the finite bornology define on $\mathbb{Z}$.

To prove $\mathbb{Z}$ with finite bornology β is a bornological group $(\mathbb{Z},\beta)$.

We must prove that, the product map and the inverse map are bounded.

i. $\psi: (\mathbb{Z},\beta) \times (\mathbb{Z},\beta) \longrightarrow (\mathbb{Z},\beta)$

Let $A, B \in \beta$ be two bounded subsets, to prove that $\psi(A \times B)$ is bounded.

Thus, $\psi(A \times B) = A + B$ is bounded subset belong to $\mathbb{Z}$, since $\mathbb{Z}$ group.

Thus, the image for every two bounded sets A, B under ψ is bounded set.

ii. $\psi^{-1}: (G, \beta) \to (G, \beta)$.

Let $A \in \beta$ be a bounded set, thus $\psi^{-1}(A) = A^{-1} \subset \mathbb{Z}$, since $\mathbb{Z}$ is a group.

Then ψ^{-1} is bounded.

1.4 Some Practical Applications of Bornological Space

In this section, we recall some of the practical applications for bornological structures. In other words, how to apply the bornology to solve many problems in our live. As we know the effect of bornology is to determine the boundedness for sets, vector spaces and groups. That means, bornological structure is general solution to solve the bounded problems [2].

The most important practical application for bornology is in the spyware program KPJ. To explain how this structures, we used to solve the problems for boundedness in specific way or with more details for example in spyware program KPJ. Exactly, when they want to determine the person location, or the identity of the person from his print finger and his print eye [2].

First, assume that the person is the original point, to determine his signed or status. We start to study the (behaviour, vibrations, frequencies) of objects within his domain. That means, we study the frequency of these objects by introducing:

open unit disk B = $\{x \in V, \|x\| < 1\}$ or closed unit disk B = $\{ x \in V, \|x\| \leq 1\}$. It is an absorbent disk but we need to study the behaviour for another objects. Then, we have to define another disk B_1 that is bigger (since B in a vector space, then it is allowed for us to multiply by scalar in vector space and make B bigger), so we get another (open or closed) absorbent disk B_1 (see figure 1). Thus by the same operation we can translate to all objects. In the end, we get collection of (open or closed) disk covers the place and the finite union which it is the bigger bounded set should also be within the place not outside, that means inside the collection see [2]. (A set A is absorbent disk in a vector space V if A absorbs every subset of V consisting of a single point), i.e. absorbs every subset of V.

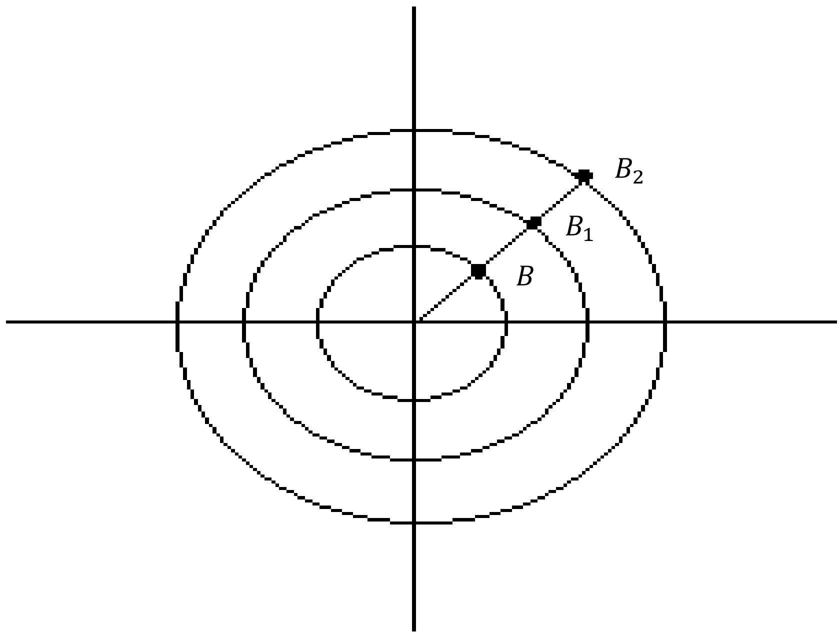

Figure1: The behaviour of objects within his domain by introducing open unit disk [2].

The fingerprint is one of the applications of the bornological space. Each person has his fingerprint that differs from the other. We start from the center, around the center there is a family of bounded sets that, the union of it, give the largest set covering the area of the thumb. Also, the hereditary property is available where a large set exists inside contains a smaller set, and so on. That means subset property is there is a transfer of all biological, formal, and life characteristics from the large set to the smaller set, and the finite union of these bounded sets gives a bounded set that belongs to the family; (see Figure 2).

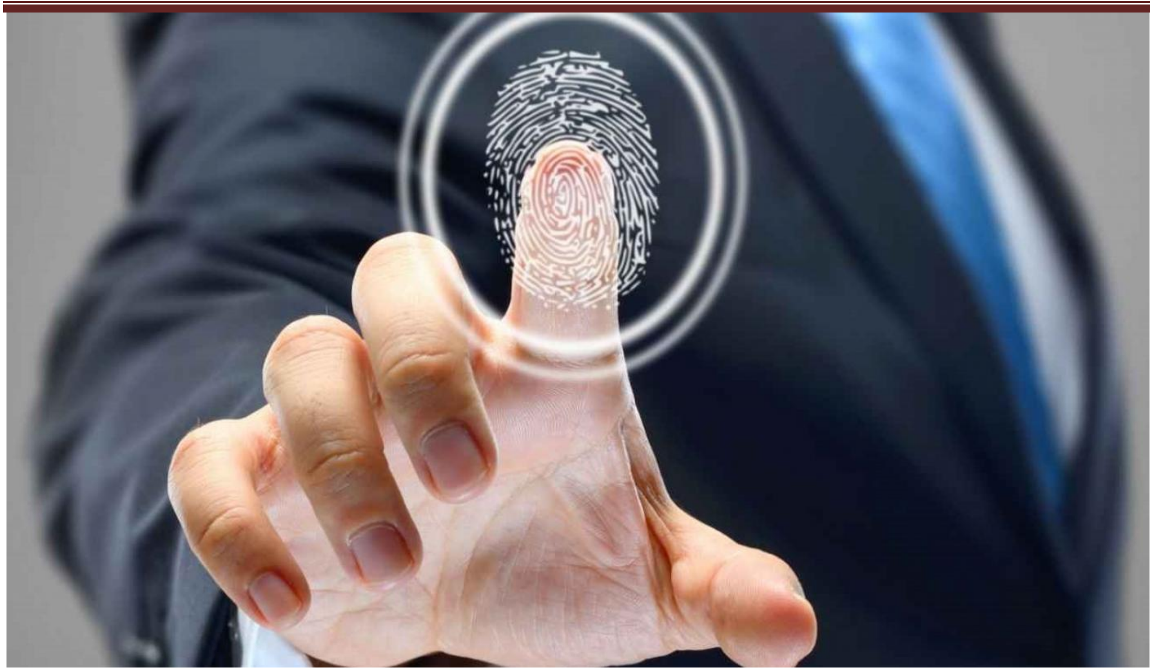

Figure 2: Finger print [23].

Another application to the iris, the pupil is the set of holes of various shapes and sizes and the distance between each other, which are located around the pupil, which acts as a (limited group) and the iris is considered one of the best ways to verify the identity of people, and no two eyes are alike in everything scientists have confirmed that it is impossible for the two eyes to be exactly the same; (see Figure 3).

Figure 3: Eye print [24].

Another application of this work is the tree trunk, where the age of the tree can be known from the number of rings in the target trunk after cutting it, the number of rings indicates the number of years the tree live; (see Figure 4).

Figure 4: The tree trunk [25].

For more information, there are many other applications of bornology, for example to equip a building with the internet service. We take the center point on the surface, and the internet waves are the bounded intervals and therefor the collection of these open intervals which it is covers the surface, also stable under finite union and hereditary property can determine the place or the building see [2]. Add to that, there is an important application of bornology, such that in satellite broadcast system when they want to determine the limits of the broadcast area; (see Figure 5).

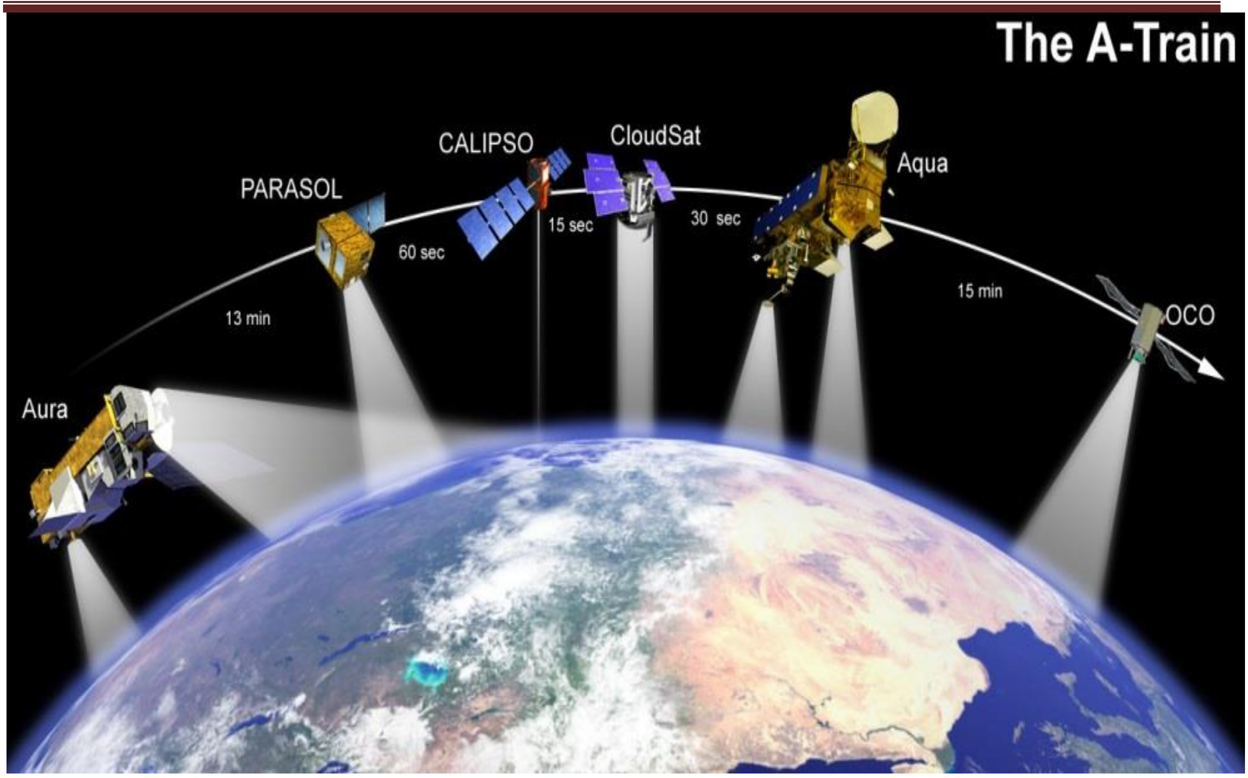

Figure 5: Satellite broadcast system [2].

1.5 Soft Set

Definition 1-5-1 [15]:

Let U be a universal set and E be a set of parameters. A ***soft set*** under U is a pair (ξ, A) consisting of a subset A of E and a mapping $\xi: A \longrightarrow P(U)$.

That mean, $\xi = \{(e, \xi(e)), e \in A, \xi(e) \in P(U)\}$.

Note that $S(U)$ is referred to the set of all soft sets over U.

Definition 1-5-2 [22]:

Suppose (ξ, A) is a soft set. A function $a: A \longrightarrow U$ is called a ***soft element*** of (ξ, A) if $a(e) \in \xi(e)$ for all $e \in A$.

Definition 1-5-3 [22]:

For each $a \in (\xi, A)$, a ***singleton soft set*** $\{a\}$ is defined by $\{a\}: A \longrightarrow P(U)$ such that $\{a\}(e) = \{a(e)\}$. Clearly a soft set (ξ, A) is singleton if (ξ, A) is a singleton set for every e. A singleton soft set contains only one soft element.

Propositions 1-5-4 [22]:

1. If $A = \emptyset$, then $\xi(e) = \emptyset$, we write $(\emptyset, \emptyset)$ is referred to describe an empty soft set, which is denoted by $\widetilde{\emptyset}$.
2. If $A = E$, then $\xi(e) = U$, we write (U, E) is referred to a universal soft set, which is denoted by $\widetilde{U}$.
3. If $L \subseteq A$, then $\xi(e) \subseteq \xi'(e)$, we write $(\xi, L) \widetilde{\subseteq} (\xi', A)$ is referred to a soft subset of a soft set.
4. If $L = A$, then $\xi(e) \subseteq \xi'(e)$ and $\xi'(e) \subseteq \xi(e)$, we write

$(\xi, L) = (\xi', A)$ is referred to soft set equal to another soft set.

5. The soft union and soft intersection of ξ and ξ' are define by the soft sets, respectively.

   $\xi(e) \cup \xi'(e), e \in L \cup A$, we write $(\xi, L) \widetilde{\cup} (\xi', A)$.

   $\xi(e) \cap \xi'(e), e \in L \cap A$, we write $(\xi, L) \widetilde{\cap} (\xi', A)$.

6. The soft complement of (ξ, A) is defined by $(\xi, A)^c = \{\xi(e)^c : e \in A\}$, where $\xi(e)^{\tilde{c}} = U \backslash \xi(e)$.
7. The composition of two soft sets is soft set.

Example 1-5-5:

Suppose that there are five cars in $U = \{c_1, c_2, c_3, c_4, c_5\}$ and

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ is a set of parameters.

e_1 = expensive, e_2 = beautiful, e_3 =sport, e_4 = cheap, e_5 = luxury,

e_6 = modern, e_7 = in good repair, e_8 = in bad repair.

Consider the functions ξ, ξ' given by ‘‘car (.)’’; (.) is to be the filled in by one of the parameters $e_i \in E$.

$$L = \{e_2, e_3, e_4, e_5, e_7\}$$

$\xi(e_2) = \{c_2, c_3, c_5\}, \xi(e_3) = \{c_2, c_4\}, \xi(e_4) = \{c_1\}, \xi(e_5) = U, \xi(e_7) = \{c_3, c_5\}$.

$$A = \{e_1, e_2, \dots, e_7\}$$

$\xi'(e_1) = \{c_3, c_5\}, \xi'(e_2) = \{c_4\}, \xi'(e_3) = \{c_2, c_4\}, \xi'(e_4) = \{c_1\}$,

$\xi'(e_5) = \{c_2, c_3, c_4, c_5\}, \xi'(e_6) = \xi'(e_7) = \{c_3\}$.

$(\xi, L) \tilde{\cap} (\xi', A) = (\xi \cap \xi', \{e_3, e_4, e_5, e_7\})$, where $(\xi \cap \xi')(e_3) = \{c_2, c_4\}$,

$(\xi \cap \xi')(e_4) = \{c_1\}, (\xi \cap \xi')(e_5) = \{c_2, c_3, c_4, c_5\}, (\xi \cap \xi')(e_7) = \{c_3\}$.

$(\xi, L) \tilde{\cup} (\xi', A) = (\xi \cup \xi', \{e_1, e_2, \dots, e_7\})$, where $(\xi \cup \xi')(e_1) = \{c_3, c_5\}$,

$(\xi \cup \xi')(e_2) = \{c_2, c_3, c_4, c_5\}, (\xi \cup \xi')(e_3) = \{c_2, c_4\}, (\xi \cup \xi')(e_4) = \{c_1\}$,

$(\xi \cup \xi')(e_5) = U, (\xi \cup \xi')(e_6) = \{c_3\}, (\xi \cup \xi')(e_7) = \{c_3, c_5\}$.

$(\xi, L)^c = (\xi^c, \{e_1, e_2, e_3, e_4, e_6, e_7, e_8\})$, where $\xi^{\tilde{c}}(e_1) = U, \xi^{\tilde{c}}(e_2) = \{c_1, c_4\}$,

$\xi^{\tilde{c}}(e_3) = \{c_1, c_3, c_5\}, \xi^{\tilde{c}}(e_4) = \{c_2, c_3, c_4, c_5\}, \xi^{\tilde{c}}(e_6) = U, \xi^{\tilde{c}}(e_7) = \{c_1, c_2, c_4\}$,

$\xi^{\tilde{c}}(e_8) = U$.

Definition 1-5-6 [14]:

Let $\xi \in S(U)$, the soft power set of ξ is characterized with $P(\xi) = \{\xi_i \tilde{\subseteq} \xi : i \in I\}$, and $|P(\xi)| = 2^{\sum_{e \in E} |\xi(e)|}$ where $|\xi(e)|$ is cardinality of $\xi(e)$.

Example 1-5-7 [14]:

Let $U = \{h_1, h_2, h_3\}$ and $A = \{e_1, e_2\}$, $\xi \in S(U)$ and

$\xi = \{(e_1, \{h_1, h_2\}), (e_2, \{h_2, h_3\})\}$, then

$$|P(\xi)| = 2^{|\xi(e_1)|+|\xi(e_2)|} = 2^{2+2} = 2^4 = 16.$$

$\xi_1 = \{(e_1, \{h_1\})\}$, $\xi_2 = \{(e_1, \{h_2\})\}$, $\xi_3 = \{(e_1, \{h_1, h_2\})\}$, $\xi_4 = \{(e_2, \{h_2\})\}$,

$\xi_5 = \{(e_2, \{h_3\})\}$, $\xi_6 = \{(e_2, \{h_2, h_3\})\}$, $\xi_7 = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$,

$\xi_8 = \{(e_1, \{h_1\}), (e_2, \{h_3\})\}$, $\xi_9 = \{(e_1, \{h_1\}), (e_2, \{h_2, h_3\})\}$,

$\xi_{10} = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$, $\xi_{11} = \{(e_1, \{h_2\}), (e_2, \{h_3\})\}$,

$\xi_{12} = \{(e_1, \{h_2\}), (e_2, \{h_2, h_3\})\}$, $\xi_{13} = \{(e_1, \{h_1, h_2\}), (e_2, \{h_2\})\}$,

$\xi_{14} = \{(e_1, \{h_1, h_2\}), (e_2, h_3)\}$, $\xi_{15} = \xi$, $\xi_{16} = \widetilde{\emptyset}$.

The soft set (ξ, A) is denoted by $\mathbb{X}$.

Definition 1-5-8 [1]:

Let G be a group and A be a nonempty subset of parameters E. For the soft set (ξ, A) over G, it is said that (ξ, A) is a ***soft group*** over G if and only if $\xi(e) \leq G, \forall\, e \in A$.

Example 1-5-9 [1]:

Assume that $G = A = S_3 = \{e, (12), (13), (23), (123), (132)\}$ and

$\xi(e) = \{e\}, \xi(12) = \{e, (12)\}, \xi(13) = \{e, (13)\}, \xi(23) = \{e, (23)\}$,

$\xi(123) = \xi(132) = \{e, (123), (132)\}$. Then (ξ, A) is a soft group over G, since $\xi(e)$ is a subgroup of G for all $e \in A$.

Definition 1-5-10 [22]:

Let $\mathbb{G} = (\xi, A)$ and $\mathbb{H} = (\xi', B)$ be two soft groups over G. Then $\mathbb{H} = (\xi', B)$ is a ***soft subgroup*** of $\mathbb{G} = (\xi, A)$ written as $(\xi', B) < (\xi, A)$ if $\xi'(e) < \xi(e)$ for all $e \in B$.

Definition 1-5-11 [19]:

Let $\mathbb{G} = (\xi, A)$ be a soft group over G and $\mathbb{H} = (\xi', B)$ is a soft subgroup of a soft group $\mathbb{G} = (\xi, A)$. Then $\mathbb{H}$ is said to be a ***normal soft subgroup*** of $\mathbb{G}$ written as $\mathbb{H} \lhd \mathbb{G}$, if $\xi'(e)$ is a normal subgroup of $\xi(e)$, for all $e \in B$.

Remarks 1-5-12 [1]:

Let G be a group and (ξ, A) is a soft group over G. Then for all $e \in A$

i. (ξ, A) is said to be identity soft group over G if $\xi(e) = \{\varepsilon\}$, where ε is the identity soft element.

ii. (ξ, A) is said to be an absolute soft group over G if $\xi(e) = G$, i.e. $A = E$.

iii. The soft group (ξ, A) is denoted by $\mathbb{G}$.

Chapter Two

Soft Bornological Set and Soft Bornological Group

2.1 Introduction

In this chapter, we construct a new structure that is called a soft bornological structure to solve the problems of boundedness for a soft set and a soft group. It is a natural to study a fundamental construction for this new structure such as soft bornological subset, product soft bornology. Also, we generate a new structure that elements are soft unbounded sets. The main important results, we prove that a family of soft bornological sets can be a partial ordered set, every soft power set of a soft set is a soft bornological set, the composition of two soft bounded maps is a soft bounded map, the intersection of two soft bornological sets is a soft bornological set, the left-right translation is a soft bornological isomorphism and the product of soft bornological groups is a soft bornological group.

2.2 Soft Bornological Set

In this section, we construct a new structure that is called a soft bornological set and discuss, its definition as well as some results.

Definition 2-2-1:

Let $\mathbb{X}$ be a nonempty soft set. A ***bornology*** on $\mathbb{X}$ is a family $\tilde{\beta} \tilde{\subseteq} P(\mathbb{X})$ such that:

i. $\tilde{\beta}$ covers $\mathbb{X}$;

ii. $\tilde{\beta}$ inclusion under hereditary. i.e. If $\tilde{B} \in \tilde{\beta}, \exists\, \tilde{A} \tilde{\subseteq} \tilde{B}$, then $\tilde{A} \in \tilde{\beta}$;

iii. $\tilde{\beta}$ inclusion under finite soft union. i.e. $\forall\, \tilde{B}_1, \tilde{B}_2 \in \tilde{\beta}$ then $\tilde{B}_1 \tilde{\cup} \tilde{B}_2 \in \tilde{\beta}$.

Then $\tilde{\beta}$ is called a soft bornology on $\mathbb{X}$ and the pair $(\mathbb{X}, \tilde{\beta})$ is a ***soft bornological set***, the elements of soft bornological set are called soft bounded subsets of $\mathbb{X}$.

Note 2-2-2:

We can satisfy the first condition in different ways, if the whole set $\mathbb{X}$ belong to the soft bornology or $\forall\, x \in \mathbb{X}$, we have $\{x\} \in \tilde{\beta}$ or $\mathbb{X} = \tilde{\bigcup}_{\tilde{B} \in \tilde{\beta}} \tilde{B}$.

Now we give the types of soft bornological sets.

1- Let $\mathbb{X}$ be a soft set, $\tilde{\beta}_{dis}$ be the collection of all soft subsets of $\mathbb{X}$, then $(\mathbb{X}, \tilde{\beta}_{dis})$ is a soft discrete bornological set.

2- Let $\mathbb{X}$ be a finite soft set, $\tilde{\beta}_{fin}$ be the collection of all finite soft bounded subsets of $\mathbb{X}$, then $(\mathbb{X}, \tilde{\beta}_{fin})$ is a soft finite bornological set.

3- Let $\mathbb{X}$ be a soft set, $\tilde{\beta}_{u}$ be the collection of all usual bounded subsets of $\mathbb{X}$, then $(\mathbb{X}, \tilde{\beta}_{u})$ is a soft usual bornological set, soft bornological set can be denoted by SBS.

Example 2-2-3:

Assume that $\mathbb{X}$ is an infinite soft set, and that $\tilde{\beta}$ is a family of all subsets of $\mathbb{X}$ which has infinite complement, such that $\tilde{\beta} = \{\tilde{B} \tilde{\subseteq} \mathbb{X}: \tilde{B}^c \ is\ an\ infinite\}$.

Then $\tilde{\beta}$ is a soft bornology on $\mathbb{X}$. Since

i. $\forall\ x \in \mathbb{X}, \{x\}$ is finite soft set belong to $\tilde{\beta}$, and $\{x\}^c$ is infinite soft set. So, $\tilde{\beta}$ covers $\mathbb{X}$.

ii. Let $\tilde{B} \in \tilde{\beta}, \tilde{A} \tilde{\subseteq} \tilde{B}$ then $\tilde{A}$ is finite soft set. (every subset of finite soft set is finite soft set), $\tilde{A}^c$ is an infinite soft set. So, $\tilde{A} \in \tilde{\beta}$. (Hereditary property).

iii. Let $\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n \in \tilde{\beta}$, then $\tilde{B}_1^c, \tilde{B}_2^c, \dots, \tilde{B}_n^c$ is infinite soft sets.

To prove that $\tilde{\bigcup}_{1 \le i \le n} \tilde{B}_i \in \tilde{\beta}$, that means.

We have to show that $(\tilde{\bigcup}_{1 \le i \le n} \tilde{B}_i)^c$ is an infinite soft set.

Since $(\tilde{\bigcup}_{1 \le i \le n} \tilde{B}_i)^c = \tilde{\bigcap}_{1 \le i \le n} \tilde{B}_i^c$ infinite soft. (By De Morgan Law)

So, $(\tilde{\bigcup}_{1 \le i \le n} \tilde{B}_i)^c$ is an infinite soft set.

Thus, $\tilde{\beta}$ inclusion under finite soft union. Then, $(\mathbb{X}, \tilde{\beta})$ is soft bornological set.

Definition 2-2-4:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set. A ***soft base*** $\tilde{\beta}_0$ is a sub collection of soft bornology $\tilde{\beta}$, and each element of the soft bornology is contained in an element of the soft base.

Example 2-2-5:

Suppose $U = \{r_1, r_2, r_3\}, A = \{e_1, e_2\}$, and
$\mathbb{X} = \{(e_1, \{r_1, r_2, r_3\}), (e_2, \{r_1, r_2, r_3\})\}$.

Then we define a soft discrete bornology on $\mathbb{X}$.

$\tilde{\beta} = \{\widetilde{\emptyset}, \mathbb{X}, \{(e_1, \{r_1\})\}, \{(e_1, \{r_2\})\}, \{(e_1, \{r_3\})\}, \{(e_1, \{r_1, r_2\})\}, \{(e_1, \{r_1, r_3\})\},$

$\{(e_1, \{r_2, r_3\})\}, \{(e_2, \{r_1\})\}, \{(e_2, \{r_2\})\}, \{(e_2, \{r_3\})\}, \{(e_2, \{r_1, r_2\})\},$

$\{(e_2, \{r_1, r_3\})\}, \{(e_2, \{r_2, r_3\})\}, \{(e_1, \{r_1\}), (e_2, \{r_1\})\}, \{(e_1, \{r_1\}), (e_2, \{r_2\})\},$

$\{(e_1, \{r_1\}), (e_2, \{r_3\})\}, \{(e_1, \{r_1\}), (e_2, \{r_1, r_2\})\}, \{(e_1, \{r_1\}), (e_2, \{r_1, r_3\})\},$

$\{(e_1, \{r_1\}), (e_2, \{r_2, r_3\})\}, \{(e_1, \{r_2\}), (e_2, \{r_1\})\}, \{(e_1, \{r_2\}), (e_2, \{r_2\})\},$

$\{(e_1, \{r_2\}), (e_2, \{r_3\})\}, \{(e_1, \{r_2\}), (e_2, \{r_1, r_2\})\}, \{(e_1, \{r_2\}), (e_2, \{r_1, r_3\})\},$

$\{(e_1, \{r_2\}), (e_2, \{r_2, r_3\})\}, \{(e_1, \{r_3\}), (e_2, \{r_1\})\}, \{(e_1, \{r_3\}), (e_2, \{r_2\})\},$

$\{(e_1, \{r_3\}), (e_2, \{r_3\})\}, \{(e_1, \{r_3\}), (e_2, \{r_1, r_2\})\}, \{(e_1, \{r_3\}), (e_2, \{r_1, r_3\})\},$

$\{(e_1, \{r_3\}), (e_2, \{r_2, r_3\})\}, \{(e_1, \{r_1, r_2\}), (e_2, \{r_1\})\}, \{(e_1, \{r_1, r_2\}), (e_2, \{r_2\})\},$

$\{(e_1, \{r_1, r_2\}), (e_2, \{r_3\})\}, \{(e_1, \{r_1, r_2\}), (e_2, \{r_1, r_2\})\}, \{(e_1, \{r_1, r_2\}),$

$(e_2, \{r_1, r_3\})\}, \{(e_1, \{r_1, r_2\}), (e_2, \{r_2, r_3\})\}, \{(e_1, \{r_1, r_3\}), (e_2, \{r_1\})\},$

$\{(e_1, \{r_1, r_3\}), (e_2, \{r_2\})\}, \{(e_1, \{r_1, r_3\}), (e_2, \{r_3\})\}, \{(e_1, \{r_1, r_3\}), (e_2, \{r_1, r_2\})\},$

$\{(e_1,\{r_1,r_3\}),(e_2,\{r_1,r_3\})\},\{(e_1,\{r_1,r_3\}),(e_2,\{r_2,r_3\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_1\})\},$

$\{(e_1,\{r_2,r_3\}),(e_2,\{r_2\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_3\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_1,r_2\})\},$

$\{(e_1,\{r_2,r_3\}),(e_2,\{r_1,r_3\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_2,r_3\})\},\{(e_1,\{ r_1\}),(e_2,\mathrm{U})\},$

$\{(e_1,\{ r_2\}),(e_2,U)\},\{(e_1,\{ r_3\}),(e_2,U)\},\{(e_1,\{ r_1,r_2\}),(e_2,U)\},\{(e_1,U)\},$

$\{(e_2,U)\},\{(e_1,\{ r_1,r_3\}),(e_2,U)\},\{(e_1,\{ r_2,r_3\}),(e_2,U)\},\{(e_1,U),(e_2,\{r_1\})\},$

$\{(e_1,U),(e_2,\{r_2\})\},\{(e_1,U),(e_2,\{r_3\})\},\{(e_1,U),(e_2,\{r_1,r_2\})\},$

$\{(e_1,U),(e_2,\{r_1,r_3\})\},\{(e_1,U),(e_2,\{r_2,r_3\})\}\}.$

Notice that $(\mathbb{X},\tilde{\beta})$ is a soft bornology on $\mathbb{X}$. Now to find the soft base of soft bornology.

$\tilde{\beta}_0=\{\ \mathbb{X},\{(e_1,\{r_1,r_2\}),(e_2,\{r_1,r_2\})\},\{(e_1,\{r_1,r_2\}),(e_2,\{r_1,r_3\})\},$

$\{(e_1,\{r_1,r_2\}),(e_2,\{r_2,r_3\})\},\{(e_1,\{r_1,r_3\}),(e_2,\{r_1,r_2\})\},\{(e_1,\{r_1,r_3\}),(e_2,\{r_1,r_3\})\},$

$\{(e_1,\{r_1,r_3\}),(e_2,\{r_2,r_3\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_1,r_2\})\},\{(e_1,\{r_2,r_3\}),(e_2,\{r_1,r_3\})\},$

$\{(e_1,\{r_2,r_3\}),(e_2,\{r_2,r_3\})\},\{(e_1,\{ r_1\}),(e_2,U)\},\{(e_1,\{ r_2\}),(e_2,U)\},$

$\{(e_1,\{ r_3\}),(e_2,U)\},\{(e_1,\{ r_1,r_2\}),(e_2,U)\},\{(e_1,\{ r_1,r_3\}),(e_2,U)\},$

$\{(e_1,\{ r_2,r_3\}),(e_2,U)\},\{(e_1,U),(e_2,\{r_1\})\},\{(e_1,U),(e_2,\{r_2\})\},$

$\{(e_1,U),(e_2,\{r_3\})\},\{(e_1,U),(e_2,\{r_1,r_2\})\},\{(e_1,U),(e_2,\{r_1,r_3\})\},$

$\{(e_1,U),(e_2,\{r_2,r_3\})\}\}$, or $\tilde{\beta}_0=\{\ \mathbb{X}\ \}$.

Definition 2-2-6:

Let $(\mathbb{X},\tilde{\beta})$ be a soft bornological set, a family $\mathbb{S}$ of members of a soft bornology $\tilde{\beta}$ is said to be ***soft subbase*** for $\tilde{\beta}$ if the family of all finite unions of members of $\mathbb{S}$ is a soft base for $\tilde{\beta}$.

Theorem 2-2-7:

Let $\mathbb{X} \neq \emptyset$, then a family $\mathbb{S}$ of subsets of $\mathbb{X}$, such that $\tilde{\bigcup}_{s \in \mathbb{S}} s$ forms a soft subbase for a soft bornology $\tilde{\beta}$ on $\mathbb{X}$.

Proof : To prove this, we must show that the family

$\tilde{\beta}_0 = \{\tilde{B} \tilde{\subseteq} \mathbb{X}: \tilde{B} = \tilde{\bigcup}_{fin} s\, , s \in \mathbb{S}\}$ forms a soft base for a soft bornology on $\mathbb{X}$.

Therefore, we must satisfy the following:

i. $\forall\, s \in \mathbb{S} \Longrightarrow\ s \tilde{\subset} \mathbb{X} \Longrightarrow s = s \,\tilde{\cup}\ s$

$$\Longrightarrow s \in \tilde{\beta}_0$$

$$\Longrightarrow \mathbb{S} \tilde{\subseteq} \tilde{\beta}_0.$$

Thus, $\mathbb{S}$ cover $\mathbb{X}$, then $\tilde{\beta}_0$ covers $\mathbb{X}$.

ii. If $\tilde{B}_1, \tilde{B}_2 \in\ \tilde{\beta}_0$

$$\Longrightarrow \tilde{B}_1 = \tilde{\bigcup}_{fin} s_i\, , \tilde{B}_2 = \tilde{\bigcup}_{fin} s_j$$

$$\tilde{B}_1\ \tilde{\cup}\ \tilde{B}_2 = \tilde{\bigcup}_{fin} s_k \ \Longrightarrow \tilde{B}_1 \,\tilde{\cup}\ \tilde{B}_2 \in\ \tilde{\beta}_0$$

That means there is $\tilde{\beta}$ on $\mathbb{X}$ for which $\tilde{\beta}_0$ form a soft base for $\tilde{\beta}$.

Thus $\tilde{\beta}_0$ forms a soft base for a soft bornology on $\mathbb{X}$. By definition of $\tilde{\beta}_0$, $\mathbb{S}$ form a soft subbase for $\tilde{\beta}$.

Example 2-2-8:

$U = \{2,4\}, A = \{e_1, e_2\}, \mathbb{X} = \{(e_1, \{2,4\}), (e_2, \{2,4\})\}$, define a soft discrete bornology $\tilde{\beta}$ on $\mathbb{X}$.

$\tilde{\beta} = \{\tilde{\emptyset}, \mathbb{X}, \{(e_1,\{2\})\}, \{(e_1,\{4\})\}, \{(e_2,\{2\})\}, \{(e_2,\{4\})\}, \{(e_1,U)\}, \{(e_2,U)\},$

$\{(e_1,\{2\}),(e_2,U)\}, \{(e_1,\{4\}),(e_2,U)\}, \{(e_1,\{2\}),(e_2,\{2\})\}, \{(e_1,\{2\}),(e_2,\{4\})\},$

$\{(e_1,\{4\}),(e_2,\{2\})\}, \{(e_1,\{4\}),(e_2,\{4\})\}, \{(e_1,U),(e_2,\{2\})\}, \{(e_1,U),(e_2,\{4\})\}\}.$

$\mathbb{S} = \{\{(e_1,\{2\})\}, \{(e_1,\{4\})\}, \{(e_2,\{2\})\}, \{(e_2,\{4\})\}\}.$

Then $\mathbb{S}$ is a soft subbase for $\tilde{\beta}$ since

$\tilde{\beta}_0 = \{\mathbb{X}, \{(e_1,U)\}, \{(e_2,U)\}, \{(e_1,\{2\}),(e_2,U)\}, \{(e_1,\{4\}),(e_2,U)\},$

$\{(e_1,U),(e_2,\{2\})\}, \{(e_1,U),(e_2,\{4\})\}\}$, or $\tilde{\beta}_0 = \{\mathbb{X}\}$ are soft bases for $\tilde{\beta}$.

By the following theorem, we prove that a family of soft bornological sets can be partial ordered set by a partial ordering relation on nonempty soft set.

<u>Theorem 2-2-9:</u>

Let $\{\tilde{\beta}_m\}_{m\in I}$ refers to a collection of all soft bornological sets on $\mathbb{X}$, $'' \preccurlyeq ''$ is a partial ordering relation if

$$(\tilde{\beta}_m \preccurlyeq \tilde{\beta}_n) \leftrightarrow (\forall \tilde{B}_m \in \tilde{\beta}_m \Rightarrow \tilde{B}_m \in \tilde{\beta}_n) \text{ for all } m,n \in I.$$

Proof:

i. Since $\tilde{\beta}_m \preccurlyeq \tilde{\beta}_m$, then $\tilde{\beta}_m \tilde{\subseteq} \tilde{\beta}_m$ for all $m \in I$. So, $'' \preccurlyeq ''$ is reflexive.

ii. Suppose that $\tilde{\beta}_m \preccurlyeq \tilde{\beta}_n$ and $\tilde{\beta}_n \preccurlyeq \tilde{\beta}_m$ for all $m,n \in I$. Then $\tilde{\beta}_m \tilde{\subseteq} \tilde{\beta}_n$ and $\tilde{\beta}_n \tilde{\subseteq} \tilde{\beta}_m$. So, $\tilde{\beta}_m = \tilde{\beta}_n$ and $'' \preccurlyeq ''$ is anti-symmetric.

iii. Suppose that $\tilde{\beta}_m \preccurlyeq \tilde{\beta}_n$ and $\tilde{\beta}_n \preccurlyeq \tilde{\beta}_k$ for all $m,n,k \in I$.

Then $\tilde{\beta}_m \tilde{\subseteq} \tilde{\beta}_n$ and $\tilde{\beta}_n \tilde{\subseteq} \tilde{\beta}_k$. So, $\tilde{\beta}_m \tilde{\subseteq} \tilde{\beta}_k$.

It implies that $\tilde{\beta}_m \preccurlyeq \tilde{\beta}_k$ and $'' \preccurlyeq ''$ has transitive property. Then $(\{\tilde{\beta}_m\}_{m\in I}, \preccurlyeq)$ is partial order set.

Proposition 2-2-10:

Every soft power set of a soft set is soft bornological set over the soft set.

Proof:

Since the soft power set of $\tilde{B}$ is $P(\tilde{B})$.

i. Since $\tilde{B} \in P(\tilde{B})$, then $P(\tilde{B})$ covers $\tilde{B}$.

ii. If $\tilde{A} \tilde{\subseteq} \tilde{B}$ and $\tilde{B} \in P(\tilde{B})$, then $\tilde{A} \in P(\tilde{B})$. (Hereditary property).

iii. Let $\tilde{A}_i \in P(\tilde{B})$, for all $i = 1, 2, \dots, n$ and since (union of finite soft set is also finite soft set)

$\tilde{\bigcup}_{1 \le i \le n} \tilde{A}_i$ is soft set.

And $\tilde{\bigcup}_{1 \le i \le n} \tilde{A}_i \in P(\tilde{B})$.

It follows that $\tilde{\bigcup}_{1 \le i \le n} \tilde{A}_i \in P(\tilde{B})$.

Then $P(\tilde{B})$ is soft bornology on $\tilde{B}$. Hence $\left(\tilde{B}, P(\tilde{B})\right)$ is called a soft bornological set.

Definition 2-2-11:

Let $(\mathbb{X}, \tilde{\beta}), (\mathbb{Y}, \tilde{\beta}')$ are soft bornological sets the function $\psi: (\mathbb{X}, \tilde{\beta}) \longrightarrow (\mathbb{Y}, \tilde{\beta}')$ is said to be a soft bounded map if the image for every soft bounded set in $(\mathbb{X}, \tilde{\beta})$ is soft bounded set in $(\mathbb{Y}, \tilde{\beta}')$. That means $\forall\, \tilde{B} \in \tilde{\beta} \Longrightarrow \psi(\tilde{B}) \in \tilde{\beta}'$.

Proposition 2-2-12:

The intersection of soft bornological sets on $\mathbb{X}$ is also soft bornological set $(\mathbb{X}, \tilde{\beta} \tilde{\cap} \tilde{\beta}')$.

Proof:

i. Since $\tilde{\beta}$ and $\tilde{\beta}'$ are soft bornologies on $\mathbb{X}$ that mean the condition of the covering is hold.

$\forall x \in \mathbb{X}, \{x\} \in \tilde{\beta}$ and $\forall x \in \mathbb{X}, \{x\} \in \tilde{\beta}'$.

That means $\forall x \in \mathbb{X}, \{x\} \in \tilde{\beta}, \tilde{\beta}'$, then $\{x\} \in \tilde{\beta} \tilde{\cap} \tilde{\beta}'$. Then $\tilde{\beta} \tilde{\cap} \tilde{\beta}'$ cover $\mathbb{X}$.

ii. Let $\tilde{B} \in \tilde{\beta} \tilde{\cap} \tilde{\beta}'$ that means $\tilde{B} \in \tilde{\beta}, \tilde{B} \in \tilde{\beta}'$.

Since $\tilde{\beta}, \tilde{\beta}'$ soft bornologies $\exists\ \tilde{A} \tilde{\subseteq}\ \tilde{B} \Longrightarrow \tilde{A} \in \tilde{\beta}, \tilde{A} \in \tilde{\beta}'$. Then $\tilde{A} \in \tilde{\beta} \tilde{\cap} \tilde{\beta}'$.

iii. Let $\tilde{A}, \tilde{B} \in \tilde{\beta} \tilde{\cap} \tilde{\beta}'$. Then $\tilde{A}, \tilde{B} \in \tilde{\beta}$ and $\tilde{A}, \tilde{B} \in \tilde{\beta}'$.

Since $\tilde{A}\ \tilde{\cup}\ \tilde{B} \in \tilde{\beta}$ and $\tilde{A}\ \tilde{\cup}\ \tilde{B} \in \tilde{\beta}'$. So, $\tilde{A}\ \tilde{\cup} \tilde{B}\ \in \tilde{\beta} \tilde{\cap} \tilde{\beta}'$.

Notice that $\tilde{\beta} \tilde{\cap} \tilde{\beta}'$ defines a soft bornology and $(\mathbb{X}, \tilde{\beta} \tilde{\cap} \tilde{\beta}')$ is soft bornological set.

By the next proposition we will prove the composition of two soft bounded maps is soft bounded map.

<u>Proposition 2-2-13:</u>

Let $\psi: (\mathbb{X}, \tilde{\beta}) \to (\mathbb{Y}, \tilde{\beta}')$ and $\phi: (\mathbb{Y}, \tilde{\beta}') \to (\mathbb{Z}, \tilde{\beta}'')$ be two soft bounded maps. Then the composition $\phi \circ \psi : (\mathbb{X}, \tilde{\beta}) \to (\mathbb{Z}, \tilde{\beta}'')$ is soft bounded map.

Proof:

Suppose $\tilde{B} \in \tilde{\beta}$ since $\psi: (\mathbb{X}, \tilde{\beta}) \to (\mathbb{Y}, \tilde{\beta}')$ is soft bounded map then $\psi(\tilde{B})$ is soft bounded set and $\psi(\tilde{B}) \in \tilde{\beta}'$. Then $\phi(\psi(\tilde{B})) \in \tilde{\beta}''$ (since ϕ is soft bounded map)

So, the composition $\phi \circ \psi: (\mathbb{X}, \tilde{\beta}) \to (\mathbb{Z}, \tilde{\beta}'')$ is soft bounded map.

The composition of three soft bounded maps is also soft bounded map and composition n of soft bounded maps is soft bounded map.

Definition 2-2-14:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set and $\mathbb{G} \tilde{\subseteq} \mathbb{X}$. The soft bornology $\tilde{\beta}_{\mathbb{G}} = \{\tilde{B} \tilde{\cap} \mathbb{G}: \tilde{B} \in \tilde{\beta}\}$ on $\mathbb{G}$ is called a soft relative bornology generated by the set $\mathbb{G}$ and the soft bornological set $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is said to a ***soft bornological subset*** of soft bornological set $(\mathbb{X}, \tilde{\beta})$.

Theorem 2-2-15:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set and $\mathbb{G} \tilde{\subseteq} \mathbb{X}$. Then the collection $\tilde{\beta}_{\mathbb{G}} = \{\tilde{B} \tilde{\cap} \mathbb{G}: \tilde{B} \in \tilde{\beta}\}$ is a soft bornology on $\mathbb{G}$ and the pair $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is a soft bornological set.

Proof:

To show that $\tilde{\beta}_{\mathbb{G}} = \{\tilde{B} \tilde{\cap} \mathbb{G}: \tilde{B} \in \tilde{\beta}\}$ is a soft bornology on $\mathbb{G}$.

i. Let $\tilde{A} = \tilde{B} \tilde{\cap} \mathbb{G}$ and $\tilde{B} \in \tilde{\beta}$, to prove that $\tilde{\beta}_{\mathbb{G}}$ is covering of $\mathbb{G}$.

i.e. $\mathbb{G} = \tilde{\bigcup}_{\tilde{A} \in \tilde{\beta}_{\mathbb{G}}} \tilde{A}$

$$\tilde{\bigcup}_{\tilde{A} \in \tilde{\beta}_{\mathbb{G}}} \tilde{A} = \tilde{\bigcup}_{\tilde{B} \in \tilde{\beta}} (\tilde{B} \tilde{\cap} \mathbb{G})$$

$$= (\tilde{\bigcup}_{\tilde{B} \in \tilde{\beta}} \tilde{B}) \tilde{\cap} \mathbb{G}$$

Where $\tilde{\bigcup}_{\tilde{B} \in \tilde{\beta}} \tilde{B} = \mathbb{X}$ and $\mathbb{X} \tilde{\cap} \mathbb{G} = \mathbb{G}$.

Then $\mathbb{G} = \tilde{\bigcup}_{\tilde{A} \in \tilde{\beta}_{\mathbb{G}}} \tilde{A}$, then $\tilde{\beta}_{\mathbb{G}}$ is covering of $\mathbb{G}$.

ii. Let $\tilde{A} \in \tilde{\beta}_{\mathbb{G}}$, i.e. $\tilde{A} = \tilde{B} \tilde{\cap} \mathbb{G}, \tilde{B} \in \tilde{\beta}$.

We take $\tilde{L} \tilde{\subseteq} \tilde{A} \Longrightarrow \tilde{L} \tilde{\subseteq} \tilde{B} \tilde{\cap} \mathbb{G}, \tilde{L} \tilde{\subseteq} \tilde{A}, \tilde{L} \tilde{\subseteq} \mathbb{G}$.

Since $\tilde{B} \in \tilde{\beta} \Longrightarrow \tilde{L} \in \tilde{\beta}$.

To prove that $\tilde{L} \in \tilde{\beta}_{\mathbb{G}}$ that means $\tilde{L} = \tilde{U} \tilde{\cap} \mathbb{G}$ where $\tilde{U} \in \tilde{\beta}$

Take $\tilde{L} = \tilde{U}$. Then $\tilde{L} = \tilde{U} \tilde{\cap} \mathbb{G}$ and $\tilde{U} \in \tilde{\beta}$. Then $\tilde{L} \in \tilde{\beta}_{\mathbb{G}}$.

iii. Let $\{\tilde{A}_i\ i \in I, I\ finite\}$, be a finite soft bounded of $\tilde{\beta}_{\mathbb{G}}$.

To prove $\tilde{\beta}_{\mathbb{G}}$ inclusion under finite soft union.

Since $\forall\, i \in I, I\ finite, \exists\, \tilde{B}_i \in \tilde{\beta}$ such that $\tilde{A}_i = \tilde{B}_i \tilde{\cap} \mathbb{G}$

$$\tilde{\bigcup_{i\in I}} \tilde{A}_i = \tilde{\bigcup_{i\in I}} (\tilde{B}_i \tilde{\cap} \mathbb{G}) = \left(\tilde{\bigcup_{i\in I}} \tilde{B}_i\right) \tilde{\cap} \mathbb{G} = \tilde{B} \tilde{\cap} \mathbb{G}, i \in I, I\ finite$$

(Since $\tilde{\beta}$ inclusion under finite soft union).

Then, $\tilde{B} \tilde{\cap} \mathbb{G} \in \tilde{\beta}_{\mathbb{G}}$. That mean $\tilde{\bigcup_{i\in I}} \tilde{A}_i \in \tilde{\beta}_{\mathbb{G}}\ , I\ finite$.

Example 2-2-16:

Consider $U = \{1,2\}, A = \{e_1, e_2\}, \mathbb{X} = \{(e_1, \{1,2\}), (e_2, \{1,2\})\}$.

Define a soft finite bornology on $\mathbb{X}$.

$\tilde{\beta} = \{\tilde{\emptyset}, \mathbb{X}, \{(e_1, \{1\})\}, \{(e_1, \{2\})\}, \{(e_2, \{1\})\}, \{(e_2, \{2\})\}, \{(e_1, U)\}, \{(e_2, U)\},$

$\{(e_1, \{1\}), (e_2, U)\}, \{(e_1, \{2\}), (e_2, U)\}, \{(e_1, \{1\}), (e_2, \{1\})\}, \{(e_1, \{1\}), (e_2, \{2\})\},$

$\{(e_1, \{2\}), (e_2, \{1\})\}, \{(e_1, \{2\}), (e_2, \{2\})\}, \{(e_1, U), (e_2, \{1\})\}, \{(e_1, U), (e_2, \{2\})\}\}$.

Let $\mathbb{G} \tilde{\subseteq} \mathbb{X}, \mathbb{G} = \{(e_1, \{1\}), (e_2, \{2\})\}$ by definition of soft bornological subset of soft bornology $\tilde{B} \tilde{\cap} \mathbb{G}, \tilde{B} \in \tilde{\beta}$.

$\tilde{\beta}_{\mathbb{G}} = \{\tilde{\emptyset} \tilde{\cap} \mathbb{G}, \{(e_1, \{1\})\} \tilde{\cap} \mathbb{G}, \{(e_1, \{2\})\} \tilde{\cap} \mathbb{G}, \{(e_2, \{1\})\} \tilde{\cap} \mathbb{G},, \{(e_2, \{2\})\} \tilde{\cap} \mathbb{G},$

$\{(e_1, U)\} \tilde{\cap} \mathbb{G}, \{(e_2, U)\} \tilde{\cap} \mathbb{G}, \{(e_1, \{1\}), (e_2, U)\} \tilde{\cap} \mathbb{G}, \{(e_1, \{2\}), (e_2, U)\} \tilde{\cap} \mathbb{G},$

$\{(e_1, \{1\}), (e_2, \{1\})\} \tilde{\cap} \mathbb{G}, \{(e_1, \{1\}), (e_2, \{2\})\} \tilde{\cap} \mathbb{G}, \{(e_1, \{2\}), (e_2, \{1\})\} \tilde{\cap} \mathbb{G},$

$\{(e_1, \{2\}), (e_2, \{2\})\} \tilde{\cap} \mathbb{G}, \{(e_1, U), (e_2, \{1\})\} \tilde{\cap} \mathbb{G}, \{(e_1, U), (e_2, \{2\})\} \tilde{\cap} \mathbb{G}, \mathbb{X} \tilde{\cap} \mathbb{G}\}.$

Then, $\tilde{\beta}_{\mathbb{G}} = \{\tilde{\emptyset}, \mathbb{G}, \{(e_1, \{1\})\}, \{(e_2, \{2\})\}\}$.

Lemma 2-2-17:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set and $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ be a soft bornological subset of soft bornological set $(\mathbb{X}, \tilde{\beta})$. $\mathbb{H}, \mathbb{G} \in \tilde{\beta}$ if $\mathbb{H} \tilde{\subseteq} \mathbb{G}$, then $\mathbb{H} \in \tilde{\beta}_{\mathbb{G}}$.

Proof:

Suppose that $\mathbb{H} \in \tilde{\beta}$. Since $\mathbb{H} \tilde{\subseteq} \mathbb{G}$, $\mathbb{H} = \mathbb{H} \tilde{\cap} \mathbb{G}$ then $\mathbb{H} \in \tilde{\beta}_{\mathbb{G}}$ by assumption $\mathbb{H} \tilde{\subseteq} \mathbb{G}$ and $\mathbb{H} \in \tilde{\beta}$.

Theorem 2-2-18:

Let $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ be a soft bornological subset of soft bornological set $(\mathbb{X}, \tilde{\beta})$. Then the following are equivalent:

i. $\mathbb{G} \in \tilde{\beta}$.

ii. $\tilde{\beta}_{\mathbb{G}} \tilde{\subseteq} \tilde{\beta}$.

Proof:

$(i) \longrightarrow (ii)$ Let $\mathbb{G} \in \tilde{\beta}$, and $\mathbb{H} \in \tilde{\beta}_{\mathbb{G}}$.

From the definition of soft bornological subset, $\mathbb{H} = \tilde{m} \tilde{\cap} \mathbb{G}$, where $\tilde{m} \in \tilde{\beta}$. Since $\mathbb{G} \in \tilde{\beta}, \tilde{m} \in \tilde{\beta}$ then, $\mathbb{H} \in \tilde{\beta}$.

$(ii) \longrightarrow (i)$ Assume that $\tilde{\beta}_{\mathbb{G}} \tilde{\subseteq} \tilde{\beta}$. Since $\mathbb{G} \in \tilde{\beta}_{\mathbb{G}}$ then, $\mathbb{G} \in \tilde{\beta}$.

Definition 2-2-19:

Let $\mathbb{H} \tilde{\subseteq} \mathbb{G} \tilde{\subseteq} \mathbb{X}$, $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ and $(\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ be soft bornological subsets of soft bornological set of $(\mathbb{X}, \tilde{\beta})$, and $\left(\mathbb{H}, \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}\right) \tilde{\subseteq} \left(\mathbb{G}, \tilde{\beta}_{\mathbb{G}}\right)$. Then, $\left(\mathbb{H}, \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}\right)$ is called a ***soft bornological subset of a soft bornological subset*** $\left(\mathbb{G}, \tilde{\beta}_{\mathbb{G}}\right)$.

Theorem 2-2-20:

Let $\mathbb{H} \tilde{\subseteq} \mathbb{G} \tilde{\subseteq} \mathbb{X}$, $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ and $(\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ be soft bornological subsets of soft bornological set $(\mathbb{X}, \tilde{\beta})$ and $(\mathbb{H}, \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}) \tilde{\subseteq} (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$. Then, $\tilde{\beta}_{\mathbb{H}} = (\tilde{\beta}_{\mathbb{G}})_{\mathbb{H}}$.

Proof:

$\tilde{\beta}_{\mathbb{H}} = \{\tilde{B} \tilde{\cap} \mathbb{H}: \tilde{B} \in \tilde{\beta}\}$ and $\tilde{\beta}_{\mathbb{G}} = \{\tilde{B} \tilde{\cap} \mathbb{G}: \tilde{B} \in \tilde{\beta}\}$. Therefore

$$\begin{aligned} \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}} &= \{\left(\tilde{B} \tilde{\cap} \mathbb{G}\right) \tilde{\cap} \mathbb{H}: \tilde{B} \in \tilde{\beta}\} \\ &= \{\tilde{B} \tilde{\cap} (\mathbb{G} \tilde{\cap} \mathbb{H}): \tilde{B} \in \tilde{\beta}\} \\ &= \{\tilde{B} \tilde{\cap} \mathbb{H}: \tilde{B} \in \tilde{\beta}\}. \text{ Since } (\mathbb{H} \tilde{\subseteq} \mathbb{G}). \\ &= \tilde{\beta}_{\mathbb{H}}. \end{aligned}$$

Another proof:

Assume $\forall\, \tilde{w} \in \tilde{\beta}_{\mathbb{H}}$. From the definition of soft bornology subset,

$\tilde{w} = \tilde{w} \tilde{\cap} \mathbb{H}$, where $\tilde{w} \in \tilde{\beta}$.

We obtain that $\tilde{w} \tilde{\cap} \mathbb{G} \in \tilde{\beta}_{\mathbb{G}}$. Then, by choosing $\tilde{w} \tilde{\cap} \mathbb{G} = \tilde{Y}$

$\tilde{Y} \tilde{\cap} \mathbb{H} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$ since of $(\mathbb{H}, \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}) \tilde{\subseteq} (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ and $\tilde{Y} \in \tilde{\beta}_{\mathbb{G}}$.

But $\tilde{Y} = \tilde{w} \tilde{\cap} \mathbb{G}$ then, $\tilde{Y} \tilde{\cap} \mathbb{H} = \tilde{w} \tilde{\cap} \mathbb{G} \tilde{\cap} \mathbb{H} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$.

$\mathbb{H} \tilde{\subseteq} \mathbb{G} \Leftrightarrow \mathbb{H} = \mathbb{H} \tilde{\cap} \mathbb{G}$ then,

$\tilde{Y} \tilde{\cap} \mathbb{H} = \tilde{w} \tilde{\cap} \mathbb{H} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$. Since $\tilde{w} = \tilde{w} \tilde{\cap} \mathbb{H} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$ then,

$\tilde{w} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$. Hence, we get $\tilde{\beta}_{\mathbb{H}} \tilde{\subseteq} \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$.

Conversely, assume that $\forall \tilde{Z} \in \left(\tilde{\beta}_{\mathbb{G}}\right)_{\mathbb{H}}$. From the definition of soft bornology subset,

$\tilde{Z} = \tilde{T} \tilde{\cap} \mathbb{H}$, where $\tilde{T} \in \tilde{\beta}_{\mathbb{G}}$. We obtain that $\tilde{T} = \tilde{w} \tilde{\cap} \mathbb{G}$, where $\tilde{w} \in \tilde{\beta}$.

$\tilde{Z} = \tilde{T} \tilde{\cap} \mathbb{H} = \tilde{w} \tilde{\cap} \mathbb{G} \tilde{\cap} \mathbb{H} = \tilde{w} \tilde{\cap} \mathbb{H} \in \tilde{\beta}_{\mathbb{H}}$. So, this completes the proof.

Definition 2-2-21:

Let $\{\tilde{\beta}_m\}_{m\in I}$ be a family of all soft bornological sets on $\mathbb{X}$. If $\tilde{\beta}_m \leq \tilde{\beta}_n$ then $\tilde{\beta}_m$ is soft finer than $\tilde{\beta}_n$. In this case $\tilde{\beta}_n$ is said to be soft coarser than $\tilde{\beta}_m$.

Example 2-2-22:

Let $\mathbb{X}$ be an infinite soft set, $\tilde{\beta}_{fin}$ is a collection of all finite soft subsets of $\mathbb{X}$, and $\tilde{\beta}_{dis}$ is a collection of all soft subsets of $\mathbb{X}$. Then $\tilde{\beta}_{fin}$ is a finer soft bornology than soft discrete bornology $\tilde{\beta}_{dis}$ on $\mathbb{X}$

$$\tilde{\beta}_{fin} \tilde{\subset} \tilde{\beta}_{dis}.$$

Proposition 2-2-23:

If $(\mathbb{F}, \tilde{\beta}_{\mathbb{F}})$ and $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ are soft bornological subsets of soft bornological sets $(\mathbb{X}, \tilde{\beta})$ and $(\mathbb{Y}, \tilde{\beta}')$, respectively and ψ is soft bounded mapping from $(\mathbb{X}, \tilde{\beta})$ into $(\mathbb{Y}, \tilde{\beta}')$ such that $\psi(\mathbb{F}) \tilde{\subseteq} \mathbb{G}$. Then ψ is relatively soft bounded mapping of $(\mathbb{F}, \tilde{\beta}_{\mathbb{F}})$ into $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$.

Proof:

Let $\tilde{A}' \in \tilde{\beta}_{\mathbb{F}}$, then there is $\tilde{A} \in \tilde{\beta}$. $\tilde{A}' = \tilde{A} \tilde{\cap} \mathbb{F}$, $\psi(\tilde{A}) \in \tilde{\beta}'$, hence

$$\psi(\tilde{A}') \tilde{\cap} \mathbb{G} = \psi(\tilde{A} \tilde{\cap} \mathbb{F}) \tilde{\cap} \mathbb{G}$$

$$= \psi(\tilde{A}) \tilde{\cap} \psi(\mathbb{F}) \tilde{\cap} \mathbb{G} \in \tilde{\beta}_{\mathbb{G}}.$$

(since $\psi(\mathbb{F}) \tilde{\subseteq} \mathbb{G}$).

So, $\psi: (\mathbb{F}, \tilde{\beta}_{\mathbb{F}}) \to (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is relatively soft bounded mapping.

Proposition 2-2-24:

Let $(\mathbb{F}, \tilde{\beta}_{\mathbb{F}}), (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$, and $(\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ be soft bornological subsets of soft bornological sets $(\mathbb{X}, \tilde{\beta}), (\mathbb{Y}, \tilde{\beta}'), (\mathbb{Z}, \tilde{\beta}'')$ respectively. $\psi: (\mathbb{F}, \tilde{\beta}_{\mathbb{F}}) \to (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$, $\phi: (\mathbb{G}, \tilde{\beta}_{\mathbb{G}}) \to (\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ are two relatively soft bounded maps. If the image for every soft bounded in $\tilde{\beta}_{\mathbb{F}}$ is soft bounded set in $\tilde{\beta}_{\mathbb{G}}$, then the composition $\phi \circ \psi: (\mathbb{F}, \tilde{\beta}_{\mathbb{F}}) \to (\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ is relatively soft bounded mapping.

Proof:

Let $\tilde{A} \in \tilde{\beta}_{\mathbb{F}}$ and since $\psi: (\mathbb{F}, \tilde{\beta}_{\mathbb{F}}) \to (\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ relatively soft bounded mapping

It follows that

$$\psi(\tilde{A}) \tilde{\cap} \mathbb{G} \in \tilde{\beta}_{\mathbb{G}}.$$

Also

$$\phi: (\mathbb{G}, \tilde{\beta}_{\mathbb{G}}) \to (\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$$

is relatively soft bounded mapping.

It follows that

$$\phi\big(\psi(\tilde{A}) \tilde{\cap} \mathbb{G}\big) \tilde{\cap} \mathbb{H} \in \tilde{\beta}_{\mathbb{H}} \quad \text{(since } \psi(\tilde{A}) \tilde{\subseteq} \mathbb{G})$$

$$\phi\left(\psi\left(\tilde{A}\right)\right) \tilde{\cap} \mathbb{H} \in \tilde{\beta}_{\mathbb{H}}$$

$$\left[(\phi \circ \psi)\left(\tilde{A}\right)\right] \tilde{\cap} \mathbb{H} \in \tilde{\beta}_{\mathbb{H}}.$$

Thus, the composition $\phi \circ \psi: (\mathbb{F}, \tilde{\beta}_{\mathbb{F}}) \to (\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ is relatively soft bounded mapping.

Definition 2-2-25:

Let $(\mathbb{X}, \tilde{\beta})$, and $(\mathbb{Y}, \tilde{\beta}')$ be two soft bornological sets. A map $\psi: (\mathbb{X}, \tilde{\beta}) \to (\mathbb{Y}, \tilde{\beta}')$ is called a ***soft bornological isomorphism*** if ψ, and ψ^{-1} are soft bounded maps and bijective.

Example 2-2-26:

$U = \{5,6\}, U' = \{10,12\}, A = \{e\}, \mathbb{X} = \{(e, \{5,6\})\}, \mathbb{Y} = \{(e, \{10,12\})\}$.

With soft finite bornology on $\mathbb{X}$ and $\mathbb{Y}$.

$\tilde{\beta} = \{\tilde{\emptyset}, \mathbb{X}, \{(e, \{5\})\}, \{(e, \{6\})\}\}$,

$\tilde{\beta}' = \{\tilde{\emptyset}, \mathbb{Y}, \{(e, \{10\})\}, \{(e, \{12\})\}\}$.

Define a map $\psi: (\mathbb{X}, \tilde{\beta}) \longrightarrow (\mathbb{Y}, \tilde{\beta}')$ by

$$\psi(\{(e, \{5\})\}) = \{(e, \{10\})\}$$

$$\psi(\{(e, \{6\})\}) = \{(e, \{12\})\}.$$

It is clear that ψ is bijective so ψ and ψ^{-1} are soft bounded maps.

$$\psi^{-1}(\{(e, \{10\})\}) = \{(e, \{5\})\}$$

$$\psi^{-1}(\{(e, \{12\})\}) = \{(e, \{6\})\}$$

$$\psi^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$$

$$\psi^{-1}(\mathbb{Y}) = \mathbb{X}.$$

Then, ψ is a soft bornological isomorphism.

Definition 2-2-27:

Let $(\mathbb{X}, \tilde{\beta})$, and $(\mathbb{Y}, \tilde{\beta}')$ be two soft bornological sets, and $\tilde{\beta}_0 =$ {The family of all $\tilde{B} \times \tilde{B}'$ where $\tilde{B} \in \tilde{\beta}, \tilde{B}' \in \tilde{\beta}'$}. We say that $\tilde{\beta}_0$ is a soft base, and is called ***product soft bornology***.

If we defines a soft bornological structure on $\mathbb{X} \times \mathbb{Y}$. Then the product sets $\mathbb{X} \times \mathbb{Y}$ with this soft bornological structure is called a soft bornological product sets of $(\mathbb{X}, \tilde{\beta})$, and $(\mathbb{Y}, \tilde{\beta}')$.

Example 2-2-28:

$U = \{5,6\}, U' = \{8,9\}, A = A' = \{e_1, e_2\}$,

$\mathbb{X} = \{(e_1, \{5,6\}), (e_2, \{5,6\})\}, \mathbb{Y} = \{(e_1, \{8,9\}), (e_2, \{8,9\})\}$.

With a soft discrete bornology on $\mathbb{X}, \mathbb{Y}$.

$\tilde{\beta}' = \{\tilde{\emptyset}, \mathbb{X}, \{(e_1, \{5\})\}, \{(e_1, \{6\})\}, \{(e_2, \{5\})\}, \{(e_2, \{6\})\}, \{(e_1, U)\}, \{(e_2, U)\},$

$\{(e_1, \{5\}), (e_2, U)\}, \{(e_1, \{6\}), (e_2, U)\}, \{(e_1, \{5\}), (e_2, \{5\})\}, \{(e_1, \{5\}), (e_2, \{6\})\},$

$\{(e_1, \{6\}), (e_2, \{5\})\}, \{(e_1, \{6\}), (e_2, \{6\})\}, \{(e_1, U), (e_2, \{5\})\}, \{(e_1, U), (e_2, \{6\})\}\}$.

$\tilde{\beta}'' = \{\tilde{\emptyset}, \mathbb{Y}, \{(e_1, \{8\})\}, \{(e_1, \{9\})\}, \{(e_2, \{8\})\}, \{(e_2, \{9\})\}, \{(e_1, U')\}, \{(e_2, U')\},$

$\{(e_1, \{8\}), (e_2, U')\}, \{(e_1, \{9\}), (e_2, U')\}, \{(e_1, \{8\}), (e_2, \{8\})\}, \{(e_1, \{8\}), (e_2, \{9\})\},$

$\{(e_1, \{9\}), (e_2, \{8\})\}, \{(e_1, \{9\}), (e_2, \{9\})\}, \{(e_1, U'), (e_2, \{8\})\}, \{(e_1, U'), (e_2, \{9\})\}\}$

$$\xi \times \xi' : A \times A' \to P(U) \times P(U')$$

$$\tilde{\beta}_{\mathbb{X} \times \mathbb{Y}} = \{\left((e_i, e_j), \xi(e_i) \times \xi'(e_j)\right)\}.$$

Where $e_i \in A, e_j \in A', i, j = 1,2,\ \xi(e_i) \in P(U), \xi'(e_j) \in P(U')$.

Proposition 2-2-29:

Let $\{(\mathbb{X}_j,\tilde{\beta}_j), j=1,2,\dots,n\}$ be two soft bornological sets and $(\mathbb{X},\tilde{\beta})$ the product soft bornological set, let $\mathbb{G}_j$, $j=1,2,\dots,n$ be a soft bounded set over $\mathbb{X}_j$ and $\mathbb{G}$ be the product soft bounded set over $\mathbb{X}$. The induced soft bornology $\tilde{\beta}_{\mathbb{G}}$ on $\mathbb{G}$ has, as a base the set of product soft bounded sets of the form $\prod_{j=1}^{n}\tilde{B}'_j$ where $\tilde{B}'_j\in(\tilde{\beta}_j)_{\mathbb{G}_j}, j=1,2,\dots,n$.

Proof:

$$\tilde{\beta}_0=\{\prod_{j=1}^{n}\tilde{B}_j\,|\tilde{B}_j\in\tilde{\beta}_j, j=1,2,\dots,n\}$$

So the base for $\tilde{\beta}_{\mathbb{G}}$ is given by

$$(\tilde{\beta}_0)_{\mathbb{G}}=\left\{\left(\prod_{j=1}^{n}\tilde{B}_j\right)\tilde{\cap}\,\mathbb{G}\,\middle|\tilde{B}_j\in\tilde{\beta}_j, j=1,2,\dots,n\right\}$$

$$=\left\{\prod_{j=1}^{n}[\tilde{B}_j\,\tilde{\cap}\,\mathbb{G}_j]\,\middle|\tilde{B}_j\in\tilde{\beta}_j, j=1,2,\dots,n\right\}$$

$$=\left\{\prod_{j=1}^{n}\tilde{B}'_j\middle|\tilde{B}'_j=[\tilde{B}_j\,\tilde{\cap}\,\mathbb{G}_j]\in(\tilde{\beta}_j)_{\mathbb{G}_j}\right\}.$$

This product soft subset is denoted by $(\mathbb{G},\tilde{\beta}_{\mathbb{G}})=\prod_{j=1}^{n}[\mathbb{G}_j,(\tilde{\beta}_j)_{\mathbb{G}_j}]$.

Definition 2-2-30:

Let $(\mathbb{X},\tilde{\beta})$ be soft bornological set. A subset $\tilde{A}$ of $\mathbb{X}$ is called ***soft unbounded subset*** of $\mathbb{X}$ if $\tilde{A}^c\in\tilde{\beta}$.

Proposition 2-2-31:

Let $(\mathbb{X},\tilde{\beta})$ be a soft bornological set on $\mathbb{X}$ and $\tilde{\Gamma}$ denoted a family of all soft unbounded sets of $\mathbb{X}$. Then $\tilde{\Gamma}$ satisfies the three following:

i. $\tilde{B}^c=\mathbb{X}-\tilde{B}\in\tilde{\Gamma}$ for each non-empty set $\tilde{B}\,\tilde{\subseteq}\,\mathbb{X}$;

ii. If $\widetilde{K} \in \tilde{\Gamma}$ and $\widetilde{K} \widetilde{\subseteq} \tilde{P}$ then $\tilde{P} \in \tilde{\Gamma}$;

iii. The finite intersection of members of $\tilde{\Gamma}$ is also member of $\tilde{\Gamma}$.

Conversely, if $\mathbb{X} \neq \emptyset$ and $\tilde{\Gamma}$ is the collection of subsets of $\mathbb{X}$ such that $\tilde{\Gamma}$ satisfies (i, ii, iii). Then there is a soft bornology $\tilde{\beta}$ on $\mathbb{X}$ such that $\tilde{\Gamma}$ forms the set of all soft unbounded subsets of $\mathbb{X}$.

Proof:

i. Let $\tilde{B}$ be a nonempty subset of $\mathbb{X}$, i.e. $\tilde{B} = \widetilde{\bigcup_{x \in \tilde{B}}} \{x\} \in \tilde{\beta}$

Then $\tilde{B}^c = \mathbb{X} - \tilde{B} \in \tilde{\Gamma}$;

ii. Let $\widetilde{K} \in \tilde{\Gamma}$ and $\widetilde{K} \widetilde{\subseteq} \tilde{P} \Longrightarrow \tilde{P}^c \widetilde{\subseteq} \widetilde{K}^c$.

Since $\widetilde{K}^c \in \tilde{\beta}$ (by definition of $\tilde{\Gamma}$) then $\tilde{P}^c \in \tilde{\beta}$. By condition (ii) of soft bornology then $\tilde{P} \in \tilde{\Gamma}$;

iii. If $\widetilde{K}_1, \widetilde{K}_2 \in \tilde{\Gamma}$ then

$\widetilde{K}_1^c, \widetilde{K}_2^c \in \tilde{\beta}$, since $\tilde{\beta}$ is a soft bornology on $\mathbb{X}$ then

$\widetilde{K}_1^c \,\widetilde{\cup}\, \widetilde{K}_2^c = (\widetilde{K}_1 \,\widetilde{\cap}\, \widetilde{K}_2)^c \in \tilde{\beta}$ then $\widetilde{K}_1 \,\widetilde{\cap}\, \widetilde{K}_2 \in \tilde{\Gamma}$.

Conversely, we define $\tilde{\beta} = \{\tilde{B} \widetilde{\subseteq} \mathbb{X}: \tilde{B}^c \in \tilde{\Gamma}\}$.

a. By (i) $\mathbb{X} - \{x\} \in \tilde{\Gamma}$ and $\forall\, x \in \mathbb{X}, \{x\} \in \tilde{\beta}$. Then $\tilde{\beta}$ covers $\mathbb{X}$;

b. If $\tilde{B} \in \tilde{\beta}, \tilde{A} \widetilde{\subseteq} \mathbb{X}, \tilde{A} \widetilde{\subseteq} \tilde{B}$, to prove $\tilde{A} \in \tilde{\beta}$.

Since $\tilde{B}^c \widetilde{\subseteq} \tilde{A}^c$ by definition of soft bornology, $\tilde{B}^c \in \tilde{\Gamma}$

(by (ii)) $\tilde{A}^c \in \tilde{\Gamma}$ by definition of soft bornology, $\tilde{A} \in \tilde{\beta}$;

c. If $\tilde{B}_1, \tilde{B}_2 \in \tilde{\beta}$ (by (iii)) $\tilde{B}_1^c \,\widetilde{\cap}\, \tilde{B}_2^c = \left(\tilde{B}_1 \,\widetilde{\cup}\, \tilde{B}_2\right)^c \in \tilde{\Gamma}$.

By definition of soft bornology $\tilde{B}_1 \,\widetilde{\cup}\, \tilde{B}_2 \in \tilde{\beta}$. Then $\tilde{\beta}$ is soft bornology on $\mathbb{X}$.

Theorem 2-2-32:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set, $\mathbb{G} \tilde{\subseteq} \mathbb{X}$ and $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ a soft bornological subset of $\mathbb{X}$. For the some sets $\mathbb{H} \tilde{\subseteq} \mathbb{G} \tilde{\subseteq} \mathbb{X}$, the following statements are equivalent:

i. $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$

ii. $\mathbb{H} = k \tilde{\cap} \mathbb{G}$ for some $k \in \tilde{\Gamma}$.

Proof:

$(i) \Rightarrow (ii)$ Let $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$. Then, $(\mathbb{H})^{\tilde{c}}{}_{\mathbb{G}} \in \tilde{\beta}_{\mathbb{G}}$.

So, there exists $w \in \tilde{\beta}$ such that

$(\mathbb{H})^{\tilde{c}}{}_{\mathbb{G}} = w \tilde{\cap} \mathbb{G}$. We can write $\mathbb{H} = ((\mathbb{H})^{\tilde{c}}{}_{\mathbb{G}})^{\tilde{c}}{}_{\mathbb{G}} = (w \tilde{\cap} \mathbb{G})^{\tilde{c}}{}_{\mathbb{G}}$

$$= \mathbb{G} \tilde{\setminus} (w \tilde{\cap} \mathbb{G})$$

$$= \mathbb{G} \tilde{\cap} (w \tilde{\cap} \mathbb{G})^{\tilde{c}}{}_{\mathbb{X}}$$

$$= \mathbb{G} \tilde{\cap} ((w)^{\tilde{c}}{}_{\mathbb{X}} \tilde{\cup} (\mathbb{G})^{\tilde{c}}{}_{\mathbb{X}})$$

$$= (\mathbb{G} \tilde{\cap} (w)^{\tilde{c}}{}_{\mathbb{X}}) \tilde{\cup} (\mathbb{G} \tilde{\cap} (\mathbb{G})^{\tilde{c}}{}_{\mathbb{X}})$$

$$= \mathbb{G} \tilde{\cap} (w)^{\tilde{c}}{}_{\mathbb{X}}$$

Since $w \in \tilde{\beta}$ then, $(w)^{\tilde{c}}{}_{\mathbb{X}} \in \tilde{\Gamma}$. By choosing $(w)^{\tilde{c}}{}_{\mathbb{G}} = k$.

We obtain that $\mathbb{H} = k \tilde{\cap} \mathbb{G}$ for $k \in \tilde{\Gamma}$.

$(ii) \Rightarrow (i)$ Since $\mathbb{H} = k \tilde{\cap} \mathbb{G}$ then, $(\mathbb{H})^{\tilde{c}}{}_{\mathbb{G}} = (k \tilde{\cap} \mathbb{G})^{\tilde{c}}{}_{\mathbb{G}}$.

$$= \mathbb{G} \tilde{\setminus} (k \tilde{\cap} \mathbb{G})$$

$$= \mathbb{G} \tilde{\cap} (k \tilde{\cap} \mathbb{G})^{\tilde{c}}{}_{\mathbb{X}}$$

$$= \mathbb{G} \tilde{\cap} ((k)^{\tilde{c}}{}_{\mathbb{X}} \tilde{\cup} (\mathbb{G})^{\tilde{c}}{}_{\mathbb{X}})$$

$$= (\mathbb{G} \tilde{\cap} (k)^{\tilde{c}}{}_{\mathbb{X}}) \tilde{\cup} (\mathbb{G} \tilde{\cap} (\mathbb{G})^{\tilde{c}}{}_{\mathbb{X}})$$

$$= \mathbb{G} \tilde{\cap} (k)^{\tilde{c}}{}_{\mathbb{X}}$$

Moreover, by assumption $k \in \tilde{\Gamma}$, then $(k)^{\tilde{c}}{}_{\mathbb{X}} \in \tilde{\beta}$.

Hence, we get $(\mathbb{H})^{\tilde{c}}{}_{\mathbb{G}} \in \tilde{\beta}_{\mathbb{G}}$ and $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$, as required.

Corollary 2-2-33:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set, $\mathbb{G} \tilde{\subseteq} \mathbb{X}$ and $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is a soft bornological subset of $\mathbb{X}$. If $\mathbb{H} \tilde{\subseteq} \mathbb{G}, \mathbb{H} \in \tilde{\Gamma}$ then, $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$.

Proof:

Since $\mathbb{H} \tilde{\subseteq} \mathbb{G}$ then, $\mathbb{H} = \mathbb{H} \tilde{\cap} \mathbb{G}$.

By assumption $\mathbb{H} \in \tilde{\Gamma}$ and we get $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$, by Theorem 2-2-32, as required.

Corollary 2-2-34:

Let $(\mathbb{X}, \tilde{\beta})$ be a soft bornological set, $\mathbb{G} \tilde{\subseteq} \mathbb{X}$ and $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is a soft bornological subset of $\mathbb{X}$. The following statements are equivalent:

i. $\mathbb{G} \in \tilde{\Gamma}$

ii. $\tilde{\Gamma}_{\mathbb{G}} \tilde{\subseteq} \tilde{\Gamma}$.

Proof:

$(i) \rightarrow (ii)$ Let $\mathbb{G} \in \tilde{\Gamma}$ and assume $\mathbb{H} \in \tilde{\Gamma}_{\mathbb{G}}$.

So, there exists $k \in \tilde{\Gamma}$ such that $\mathbb{H} = k \tilde{\cap} \mathbb{G}$ by Theorem 2-2-32.

Moreover, by assumption $\mathbb{G} \in \tilde{\Gamma}$ then, $\mathbb{H} \in \tilde{\Gamma}$.

Thus, $\tilde{\Gamma}_{\mathbb{G}} \tilde{\subseteq} \tilde{\Gamma}$.

$(ii) \rightarrow (i)$ Since $(\mathbb{G}, \tilde{\beta}_{\mathbb{G}})$ is a soft bornology, then $\mathbb{G} \in \tilde{\Gamma}_{\mathbb{G}}$ and $\tilde{\Gamma}_{\mathbb{G}} \tilde{\subseteq} \tilde{\Gamma}$.

Then we get $\mathbb{G} \in \tilde{\Gamma}$.

2.3 Soft Bornological Group

To solve the problem of boundedness for a soft group we construct a new structure that is called a soft bornological group.

Definition 2-3-1:

Let $\mathbb{G}$ be a soft group. We say that $(\mathbb{G}, \tilde{\beta})$ is a ***soft bornological group*** if $\tilde{\beta}$ is a soft bornology on $\mathbb{G}$, and the following conditions holds:

1. The product mapping $(\mathbb{G}, \tilde{\beta}) \times (\mathbb{G}, \tilde{\beta}) \rightarrow (\mathbb{G}, \tilde{\beta})$ is soft bounded;
2. The inverse mapping $(\mathbb{G}, \tilde{\beta}) \rightarrow (\mathbb{G}, \tilde{\beta})$ is soft bounded.

Example 2-3-2:

Let $\mathbb{G}$ be absolute soft group over G, where $G = (\mathbb{Z}_3, +_3)$, $A = (\mathbb{Z}_3, +_3)$.

We can define a soft finite bornology $\tilde{\beta}$ on this soft group. Which it is the collection of all finite soft subsets of $\mathbb{G}$.

To prove $\mathbb{G}$ with soft finite bornology $\tilde{\beta}$ is a soft bornological group $(\mathbb{G}, \tilde{\beta})$.

We must prove that the product map and inverse map are soft bounded.

1) $\psi: (\mathbb{G}, \tilde{\beta}) \times (\mathbb{G}, \tilde{\beta}) \rightarrow (\mathbb{G}, \tilde{\beta})$

Let $\tilde{B}_x$, $\tilde{B}_y$ be two soft bounded sets belong to $(\mathbb{G}, \tilde{\beta})$.

Then we must proof that $\psi(\tilde{B}_x \times \tilde{B}_y)$ is soft bounded.

$\psi(\tilde{B}_x \times \tilde{B}_y) = \{\tilde{B}_x \tilde{\mp} \tilde{B}_y \tilde{\subseteq} \tilde{B}_{x+y} : \tilde{B}_x, \tilde{B}_y, \tilde{B}_{x+y} \in (\mathbb{G}, \tilde{\beta})\}$.

Since $\mathbb{G}$ is absolute soft group.

Thus, the image for every two soft bounded sets $\tilde{B}_x, \tilde{B}_y$ under ψ is soft bounded set.

2) The inverse map $\psi^{-1}: (\mathbb{G}, \tilde{\beta}) \to (\mathbb{G}, \tilde{\beta})$

Let $\tilde{B} \in \tilde{\beta}$. Then, $\tilde{B}^{-1} = \{\tilde{B}^{-1}: \tilde{B}^{-1} \in (\mathbb{G}, \tilde{\beta})\}$.

So, ψ^{-1} is soft bounded. Then $(\mathbb{G}, \tilde{\beta})$ is a soft bornological group.

Definition 2-3-3:

Let $(\mathbb{G}, \tilde{\beta})$ and $(\mathbb{G}', \tilde{\beta}')$ be two soft bornological groups. A homomorphism of soft bornological groups is a group homomorphism which is also soft bounded.

Remark 2-3-4:

If $(\mathbb{G}, \tilde{\beta})$ is a soft bornological group, then $\tilde{B} \in \tilde{\beta}$ if and only if $\tilde{B}^{-1} \in \tilde{\beta}$.

Theorem 2-3-5:

$(\mathbb{G}, \tilde{\beta})$ is a soft bornological group iff for each $g_1, g_2 \in \mathbb{G}$ and each soft bounded sets $\tilde{B}_1, \tilde{B}_2$ containing g_1, g_2 respectively, there is a soft bounded set $\tilde{B}$ contains $g_1 * {g_2}^{-1}$ in $\mathbb{G}$ such that $\tilde{B}_1 * \ {\tilde{B}_2}^{-1} \tilde{\subseteq} \tilde{B}$.

Proof:

Let $(\mathbb{G}, \tilde{\beta})$ be a soft bornological group with respect to soft bounded set, that means (g_1, g_2) is a point in $\mathbb{G} \times \mathbb{G}$, let $\tilde{B}_1, \tilde{B}_2$ two soft bounded sets containing g_1, g_2, respectively.

Then there exist a soft bounded set $\tilde{B}$ containing $\psi(g_1, g_2) = g_1 * {g_2}^{-1}$ in $\mathbb{G}$, such that $\tilde{B}_1 * {\tilde{B}_2}^{-1} \tilde{\subseteq} \tilde{B}$.

From Remark 2-3-4, ${\tilde{B}_2}^{-1}$ is soft bounded set containing ${g_2}^{-1}$, and $\tilde{B}_1 \times \tilde{B}_2$ is soft bounded set containing (g_1, g_2).

So, $\psi(\tilde{B}_1 \times \tilde{B}_2) \tilde{\subseteq} \tilde{B}$. This means that the product map is soft bounded map.

To show that inverse map ϕ is soft bounded map let $g \in \mathbb{G}$ and let $\tilde{B}$ be a soft bounded set containing g such that $\phi(g) = g^{-1}$.

Then by Remark 2-3-4, $\tilde{B}^{-1}$ is soft bounded set containing g^{-1} satisfy

$\phi(\tilde{B}) = \tilde{B}^{-1}$ which means that ϕ is soft bounded map at g, soon $\mathbb{G}$ is a soft bornological group. The convers is clear from the concept of soft bornological group.

Definition 2-3-6:

Let $\mathbb{G}$ be a soft bornological group. For $g \in \mathbb{G}$ there is a ***left translation map*** $\psi_g: \mathbb{G} \to \mathbb{G}$ defined by $\psi_g(x) = gx$ and the ***right translation map*** $\psi_g: \mathbb{G} \to \mathbb{G}$ defined by $\psi_g(x) = xg$.

Proposition 2-3-7:

Let g be an arbitrary element of soft group $\mathbb{G}$. Then left translation map $\psi_g: \mathbb{G} \to \mathbb{G}; x \longmapsto gx$ (and right translation map $\psi_g: \mathbb{G} \to \mathbb{G}; x \longmapsto xg$) is a soft bornological isomorphism.

Proof:

It is known that left translation is bijective map and since has inverse so we can only prove that left translation is a soft bounded.

For any $x, y \in \mathbb{G}$ and a soft bounded sets $\tilde{B}_x$ and $\tilde{B}_y$ containing x, y respectively such that $\tilde{B}_x \cdot \tilde{B}_y \tilde{\subseteq} \tilde{B}$ by definition of soft bornological group, there exist a soft bounded set $\tilde{B}$ contains xy.

Hence, we have $\psi_x(\tilde{B}_y) = x \cdot \tilde{B}_y \tilde{\subset} \tilde{B}_x \cdot \tilde{B}_y \tilde{\subset} \tilde{B}.$

Corollary 2-3-8:

Let $\mathbb{H} \tilde{\subseteq} \mathbb{G}$ be a soft subgroup. $\tilde{\mathbb{H}}$ contained in a soft bounded set of $\mathbb{G}$. Then $\mathbb{H}$ is soft bounded in $\mathbb{G}$.

Let $\tilde{B}$ be a soft bounded subset of $\mathbb{G}$ with $\tilde{\mathbb{H}} \tilde{\subseteq} \tilde{B}$. A soft bornological group consists of a soft group and a soft bornology $\tilde{\beta}$ on $\mathbb{G}$ since $\tilde{\beta}$ inclusion under hereditary. Then, $\mathbb{H}$ is soft bounded in $\mathbb{G}$.

Definition 2-3-9:

Let $(\mathbb{G}, \tilde{\beta})$ be soft bornological group and $\mathbb{H} \tilde{\subseteq} \mathbb{G}$. Then $(\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$

is called a ***soft bornological subgroup*** of $\mathbb{G}$ if

i. $(\mathbb{H}, *)$ is soft subgroup of a soft group $(\mathbb{G}, *)$.

ii. $(\mathbb{H}, \tilde{\beta}_{\mathbb{H}})$ is soft bornological subset of $(\mathbb{G}, \tilde{\beta})$.

Proposition 2-3-10:

Let $(\mathbb{G}, \tilde{\beta})$ be a soft bornological group if $\tilde{A}$ is soft bounded set in $\mathbb{G}$ and $\mathbb{H} \tilde{\subseteq} \mathbb{G}$ then, $\tilde{A}\mathbb{H}, \mathbb{H}\tilde{A}$ are soft bounded in $\mathbb{G}$.

Proof:

Let $\tilde{A}$ be a soft bounded set in $\mathbb{G}$ and $\mathbb{H} \tilde{\subseteq} \mathbb{G}$, since every left translation (right translation) of soft bornological group into itself is soft bornological isomorphism, by Proposition 2-3-7.

$$\mathbb{H}\tilde{A} = \tilde{\bigcup_{h \in \mathbb{H}}} \psi_h(\tilde{A}) \quad \text{and} \quad \tilde{A}\mathbb{H} = \tilde{\bigcup_{h \in \mathbb{H}}} \psi_h(\tilde{A})$$

So, if $\mathbb{G}$ is a soft bornological group then for any soft bounded set $\tilde{A}$ in $\mathbb{G}$ and any subset $\mathbb{H}$ of $\mathbb{G}$ these $\tilde{A}\mathbb{H}, \mathbb{H}\tilde{A}$ are soft bounded sets in $\mathbb{G}$.

Theorem 2-3-11:

Let $(\mathbb{G}, \tilde{\beta})$ be a soft bornological group, $\mathbb{H}$ is normal soft subgroup of $\mathbb{G}$. Then $\mathbb{G}/_{\mathbb{H}}$ is soft bornological group.

Proof:

Let $\mathbb{G}$ be a soft bornological group and $\mathbb{H}$ be a normal soft subgroup of $\mathbb{G}$.

Then $\mathbb{G}/_{\mathbb{H}}$ has a quotient soft group structure and the function $\psi: \mathbb{G} \to \mathbb{G}/_{\mathbb{H}}$,

$g \longmapsto g\mathbb{H}$, define a quotient soft bornology on $\mathbb{G}/_{\mathbb{H}}$.

Also, the quotient function ψ is soft bounded, for if $\tilde{A}$ is soft bounded in $\mathbb{G}$ then $\psi(\tilde{A}) = \tilde{A}\mathbb{H}$, by Proposition 2-3-10.

And it follows that $\psi(\tilde{A})$ is soft bounded in $\mathbb{G}/_{\mathbb{H}}$.

If μ, μ' are the multiplication in $\mathbb{G}$ and $\mathbb{G}/_{\mathbb{H}}$, and ν, ν' are the inversion in $\mathbb{G}$ and $\mathbb{G}/_{\mathbb{H}}$, respectively.

Then μ', ν' are uniquely defined by the following commutative diagrams:

$$\begin{tikzcd}
\mathbb{G} \times \mathbb{G} \arrow[r, "\mu"] \arrow[d, "\psi \times \psi"'] \arrow[dr, bend left] & \mathbb{G} \arrow[d, "\psi"] \\
\mathbb{G}/_{\mathbb{H}} \times \mathbb{G}/_{\mathbb{H}} \arrow[r, "\mu'"'] & \mathbb{G}/_{\mathbb{H}}
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbb{G} \arrow[r, "\nu"] \arrow[d, "\psi"'] \arrow[dr, bend left] & \mathbb{G} \arrow[d, "\psi"] \\
\mathbb{G}/_{\mathbb{H}} \arrow[r, "\nu'"'] & \mathbb{G}/_{\mathbb{H}}
\end{tikzcd}$$

To prove that μ', ν' are soft bounded function. By using the following diagram:

$$\begin{tikzcd} \mathbb{G} \arrow[r, "\psi"] \arrow[d, "\psi \circ \nu"'] & {}^{\mathbb{G}}\!/_{\mathbb{H}} \arrow[dl, shift right, "\varphi"'] \arrow[dl, shift left, "\nu'"] \\ {}^{\mathbb{G}}\!/_{\mathbb{H}} & \end{tikzcd}$$

$\psi \circ \nu$ is soft bounded and ψ is soft quotient function so by the universal property of ψ there exist a unique soft bounded function φ: ${}^{\mathbb{G}}\!/_{\mathbb{H}} \to {}^{\mathbb{G}}\!/_{\mathbb{H}}$ making

$\varphi \circ \psi = \psi \circ \nu$.

But $\nu' \circ \psi = \psi \circ \nu$, so $\nu' = \varphi$, and hence ν' is soft bounded.

Also by using the following diagram:

$$\begin{tikzcd} \mathbb{G} \times \mathbb{G} \arrow[r, "\psi\times\psi"] \arrow[d, "\psi \circ \mu"'] & {}^{\mathbb{G}}\!/_{\mathbb{H}} \times {}^{\mathbb{G}}\!/_{\mathbb{H}} \arrow[dl, shift right, "\varphi'"'] \arrow[dl, shift left, "\mu'"] \\ {}^{\mathbb{G}}\!/_{\mathbb{H}} & \end{tikzcd}$$

Since $\psi \times \psi$ is soft bounded and surjection function, then $\psi \times \psi$ is soft quotient function, since $\psi \circ \mu$ is soft bounded, so by universal property of $\psi \times \psi$

there exist a unique soft bounded function $\varphi' \circ (\psi \times \psi) = \psi \circ \mu$.

But μ' satisfies the condition $\mu' \circ (\psi \times \psi) = \psi \circ \mu$.

So, $\mu' = \varphi'$, hence μ' is soft bounded.

Theorem 2-3-12:

The product of soft bornological groups is soft bornological group.

Proof:

Let $\{\mathbb{G}_i\}_{i\in I}$ be a family of soft bornological groups, their product $\mathbb{G} = \prod_{i\in I} \mathbb{G}_i$ has a natural group structure (product of soft groups). And a natural soft bornology (product of soft bornologies).

By using the following commutative diagram:

$$\begin{tikzcd}
\mathbb{G} \times \mathbb{G} \arrow[r, "\mu"] \arrow[d, "pr_i \times pr_i"'] \arrow[dr, "\mu_i \circ (pr_i \times pr_i)"] & \mathbb{G} \arrow[d, "pr_i"] \\
\mathbb{G}_i \times \mathbb{G}_i \arrow[r, "\mu_i"'] & \mathbb{G}_i
\end{tikzcd}$$

Then $\mu_i \circ (pr_i \times pr_i)$ is soft bounded.

Since μ_i and $pr_i \times pr_i$ are soft bounded. Therefor $pr_i \circ \mu = \mu_i \circ (pr_i \times pr_i)$ is soft bounded.

Now, by using the following commutative diagram:

$$\begin{tikzcd}
\mathbb{G} \arrow[r, "\nu"] \arrow[d, "pr_i"'] \arrow[dr, "\nu_i \circ pr_i"] & \mathbb{G} \arrow[d, "pr_i"] \\
\mathbb{G}_i \arrow[r, "\nu_i"'] & \mathbb{G}_i
\end{tikzcd}$$

$\nu_i \circ pr_i$ is soft bounded since ν_i and pr_i are soft bounded.

Therefor $pr_i \circ \nu = \nu_i \circ pr_i$ is soft bounded.

Therefor ν is soft bounded.

So, $\mathbb{G} = \prod_{i\in I} \mathbb{G}_i$ is a soft bornological group.

The soft bornological group $\mathbb{G} = \prod_{i\in I} \mathbb{G}_i$ is called product soft bornological group.

Chapter Three

Soft Bornological Group

Acts

on Soft Bornological Set

3.1 Introduction

In this chapter, we study soft bornological group action. That means a soft bornological group acts on a soft bornological set. This process is called a soft bounded action such that the effect of the soft bounded action is to partition a soft bornological set into classes of soft orbitals. The main important result is to prove that soft bornological group action is soft bornological isomorphism.

3.2 Soft Bornological Group Acts on Soft Bornological Set

In this section, we show that the soft bornological set is partitioned into soft orbital classes by acting as a soft bornological group on the soft bornological set.

Definition 3-2-1:

A soft bornological group action is a triple $(\mathbb{G}, \mathbb{X}, \theta_e)$ where $(\mathbb{G}, \tilde{\beta})$ a soft bornological group, $(\mathbb{X}, \tilde{\beta}')$ a soft bornological set and $\theta_e: \mathbb{G} \times \mathbb{X} \to \mathbb{X}$ is a soft bounded, such that it has the following conditions:

i. $\theta_e(\varepsilon, x) = x$, for all $x \in \mathbb{X}, \varepsilon \in \mathbb{G}$, where ε the identity soft element.

ii. $\theta_e\big(g, \theta_e(h, x)\big) = \theta_e(gh, x)$, for all $g, h \in \mathbb{G}, x \in \mathbb{X}$.

Then we say that the soft bornological group $\mathbb{G}$ acts on a soft bornological set $\mathbb{X}$ and $\mathbb{X}$ is called a ***left*** $\mathbb{G}$***- soft bornological set***.

Further, the notation $g \cdot x$ (or gx) will be used for $\theta_e(g, x)$, so that (i), (ii) become $\varepsilon \cdot x = x$ and $g \cdot (h \cdot x) = (gh) \cdot x$. If $\tilde{B}$ is soft bounded subset of soft bornological group $\mathbb{G}$ and $\tilde{A}$ is a soft bounded subset of soft bornological set $\mathbb{X}$, we put $\tilde{B} \cdot \tilde{A} = \{g \cdot x : g \in \tilde{B}, x \in \tilde{A}\} \tilde{\subseteq} \mathbb{X}$.

Remark 3-2-2:

If $(\mathbb{X}, \tilde{\beta}')$ is soft bornological set. Then $(\mathbb{G}, \tilde{\beta})$ is said to be a soft bornological transformation group on $(\mathbb{X}, \tilde{\beta}')$.

Example 3-2-3:

1- Assume $\mathbb{G}$ is a soft bornological group and $\mathbb{X} = \mathbb{G}$. The product map is define by $\theta_e: \mathbb{G} \times \mathbb{G} \to \mathbb{G}$ where $\theta_e(g, x) = g \cdot x$

such that

i. $\theta_e(\varepsilon, g) = \varepsilon \cdot g = g, \forall g \in \mathbb{G}$ and ε identity soft element of $\mathbb{G}$.

ii. $\theta_e\big(g, \theta_e(g_1, g_2)\big) = \theta_e(g, g_1 g_2) = g \cdot (g_1 g_2)$

$$= (g g_1) \cdot g_2$$

$$= \theta_e(g g_1, g_2), \forall\ g, g_1, g_2 \in \mathbb{G}.$$

It is easy to say that θ_e is soft bounded action.

Thus, every soft bornological group $\mathbb{G}$ acts on itself by product map.

2- Let $\mathbb{G}$ a soft bornological group and $\mathbb{X} = \mathbb{G}$ be a soft bornological set. Define a mapping $\theta_e: \mathbb{G} \times \mathbb{G} \to \mathbb{G}$ as following $\theta_e(g, h) = g h g^{-1}, \forall g, h \in \mathbb{G}$ such that

i. $\theta_e(\varepsilon, h) = \varepsilon h \varepsilon^{-1} = h, \forall\ h, \varepsilon \in \mathbb{G}$.

ii. $\theta_e\big(g_1, \theta_e(g_2, h)\big) = \theta_e\big(g_1, (g_2 h {g_2}^{-1})\big)$

$$= g_1\ (g_2 h {g_2}^{-1}) {g_1}^{-1}$$

$$= (g_1 g_2) h ((g_1 g_2)^{-1}$$

$$= \theta_e(g_1 g_2, h).$$

Then θ_e is a soft bounded and this action is called conjugate action of $\mathbb{G}$ on itself.

Definition 3-2-4:

If $\mathbb{X}$ be a $\mathbb{G}$- soft bornological set, and $\tilde{B}$ be a soft bounded subset of $\mathbb{X}$. Then a subset $\tilde{B}$ of $\mathbb{X}$ is called an ***invariant soft bornological set*** under the actions of soft bornological group $\mathbb{G}$ if $\mathbb{G} \cdot \tilde{B} = \tilde{B}$. i.e. $\theta_e\big(\mathbb{G} \times \tilde{B}\big) = \tilde{B}$.

Example 3-2-5:

Suppose $(\mathbb{Z}, \tilde{\beta}_{fin})$ be a soft finite bornological group and $\mathbb{X} = (\mathbb{R}, \tilde{\beta}_u)$ be a soft usual bornological set. Let the soft bornological group $\mathbb{Z}$ action on the soft bornological set $\mathbb{R}$ by

$$\theta_e(z, r) = z \tilde{+} r \quad \text{for all } z \in \mathbb{Z}, r \in \mathbb{R}.$$

Then $\mathbb{R}$ is $\mathbb{Z}$- soft bornological set and $\mathbb{Q}$ the set of all rational numbers as a subset of $\mathbb{R}$ is invariant soft bornological set such that

$$\mathbb{Z} \cdot \mathbb{Q} = \{z \tilde{+} q : z \in \mathbb{Z}, q \in \mathbb{Q}\} = \mathbb{Q}.$$

Theorem 3-2-6:

The soft bounded action is soft bornological isomorphism.

Proof:

Let

$$\theta_e\colon \mathbb{G} \times \mathbb{X} \longrightarrow \mathbb{X}$$

be a soft bounded action of a soft bornological group $\mathbb{G}$ on a soft bornological set $\mathbb{X}$. Every element $g \in \mathbb{G}$ determines a soft bounded translation $(\theta_e)_g$ of $\mathbb{X}$ onto itself, defined by:

$$(\theta_e)_g(x) = (\theta_e)(g, x), \text{ for each } x \in \mathbb{X}.$$

Then by Definition (3-2-1) (ii)

$$(\theta_e)_h \circ (\theta_e)_g = (\theta_e)_{hg}$$

$$\begin{aligned}\big((\theta_e)_h \circ (\theta_e)_g\big)(x) &= (\theta_e)_h \big((\theta_e)_g\big)(x) \\ &= (\theta_e)_h(\theta_e(g, x)) \\ &= \theta_e\big(h, \theta_e(g, x)\big) \\ &= \theta_e(hg, x) \\ &= (\theta_e)_{hg}(x).\end{aligned}$$

and by Definition (3-2-1) (i), $(\theta_e)_\varepsilon = I_{\mathbb{X}}$, the identity mapping of $\mathbb{X}$ onto itself. Thus

$$(\theta_e)_g \circ (\theta_e)_{g^{-1}} = (\theta_e)_{gg^{-1}} = (\theta_e)_\varepsilon = (\theta_e)_{g^{-1}} \circ (\theta_e)_g.$$

$$\begin{aligned}
\left((\theta_e)_g \circ (\theta_e)_{g^{-1}}\right)(x) &= (\theta_e)_g \left((\theta_e)_{g^{-1}}(x)\right) \\
&= \theta_e\big(g, \theta_e(g^{-1}, x)\big) \\
&= \theta_e(gg^{-1}, x) \\
&= \theta_e(g^{-1}g, x) \\
&= \theta_e\big(g^{-1}, \theta_e(g, x)\big) \\
&= \theta_e\left(g^{-1}, (\theta_e)_g(x)\right) \\
&= (\theta_e)_{g^{-1}}\left((\theta_e)_g(x)\right) \\
&= \left((\theta_e)_{g^{-1}} \circ (\theta_e)_g\right)(x)
\end{aligned}$$

Hence, $((\theta_e)_g)^{-1} = (\theta_e)_{g^{-1}}$ is the soft bounded inverse mapping of $(\theta_e)_g$, which means that each $(\theta_e)_g$ is an isomorphism of soft bornological set $\mathbb{X}$.

Proposition 3-2-7:

Let $(\mathbb{X}, \tilde{\beta}')$ be a soft bornological set and $(\mathbb{Y}, \tilde{\beta}'')$ be a soft bornological set. If $(\mathbb{X}, \tilde{\beta}')$ and $(\mathbb{Y}, \tilde{\beta}'')$ are two $\mathbb{G}$- soft bornological set, then $(\mathbb{X} \times \mathbb{Y}, \tilde{\beta}' \times \tilde{\beta}'')$ is $\mathbb{G}$- soft bornological set.

Proof:

Suppose $(\mathbb{G}, \tilde{\beta})$ is soft bornological group. Consider $\mathbb{G}$- soft bornological set $(\mathbb{X}, \tilde{\beta}')$ with the soft (bounded) action

$$\theta_e: \mathbb{G} \times \mathbb{X} \to \mathbb{X}$$

and $\mathbb{G}$- soft bornological set $(\mathbb{Y}, \tilde{\beta}'')$ with the soft (bounded) action

$$\phi_e: \mathbb{G} \times \mathbb{Y} \to \mathbb{Y}$$

Then, we establish a soft (bounded) action denoted as

$$\varphi_e: \mathbb{G} \times (\mathbb{X} \times \mathbb{Y}) \to (\mathbb{X} \times \mathbb{Y}).$$

That means, $\big(g,(x,y)\big) \longmapsto \varphi_e\big(g,(x,y)\big) = \big(\theta_e(g,x), \phi_e(g,y)\big)$. That we want to show the following:

i. $\varphi_e\big(g_1, \varphi_e(g_2,(x,y))\big) = \varphi_e\Big(g_1, \big(\theta_e(g_2,x), \phi_e(g_2,y)\big)\Big)$

$$= \varphi_e\big(g_1, (g_2x, g_2y)\big)$$

$$= \big(\theta_e(g_1, g_2x), \phi_e(g_1, g_2y)\big)$$

$$= \big(\theta_e(g_1g_2, x), \phi_e(g_1g_2, y)\big)$$

$$= \varphi_e\big(g_1g_2, (x,y)\big).$$

$\forall\, g_1, g_2 \in \mathbb{G}, (x,y) \in \mathbb{X} \times \mathbb{Y}$.

ii. $\varphi_e\big(\varepsilon,(x,y)\big) = \big(\theta_e(\varepsilon,x), \phi_e(\varepsilon,y)\big)$

$$= (\varepsilon x, \varepsilon y)$$

$$= (x,y), \forall \varepsilon \in \mathbb{G}, (x,y) \in \mathbb{X} \times \mathbb{Y}.$$

Therefore φ_e is a soft (bounded) action of $(\mathbb{G}, \tilde{\beta})$ on $(\mathbb{X} \times \mathbb{Y}, \tilde{\beta}' \times \tilde{\beta}'')$.

<u>Proposition 3-2-8:</u>

For the $\mathbb{G}$-soft bornological set, if $x \in \mathbb{X}, g \in \mathbb{G}$, and $y = \theta_e(g,x)$,

then $x = \theta_e(g^{-1}, y)$. Moreover, if $x \neq x'$ then $\theta_e(g,x) \neq \theta_e(g,x')$.

Proof:

Let $y = \theta_e(g,x)$ such that

$$\theta_e(g^{-1}, y) = \theta_e\big(g^{-1}, \theta_e(g, x)\big) = \theta_e(g^{-1}g, x) = \theta_e(\varepsilon, x) = x.$$

Assume $x \neq x'$, and $\theta_e(g, x) = \theta_e(g, x')$. When g^{-1} is applied to both sides, it follow that

$$\theta_e\big(g^{-1}, \theta_e(g, x)\big) = \theta_e(g^{-1}, \theta_e(g, x'))$$

$$\theta_e(g^{-1}g, x) = \theta_e(g^{-1}g, x')$$

$$\theta_e(\varepsilon, x) = \theta_e(\varepsilon, x')$$

$$x = x'$$

This contradicts the hypothesis $x \neq x'$. Hence, $\theta_e(g, x) \neq \theta_e(g, x')$.

Definition 3-2-9:

A soft (bounded) action of the soft bornological group $(\mathbb{G}, \tilde{\beta})$ on the soft bornological set $(\mathbb{X}, \tilde{\beta}')$, this action is called:

1. ***Transitive*** if for each pair $x, y \in \mathbb{X}$ there exists an element g in $\mathbb{G}$ such that $\theta_e(g, x) = y$.
2. ***Effective*** (or faithful) if for each two disjoint elements $g, h \in \mathbb{G}$ there is an element $x \in \mathbb{X}$ such that $\theta_e(g, x) \neq \theta_e(h, x)$.
3. ***Free*** if give $g, h \in \mathbb{G}$, the existence of an element $x \in \mathbb{X}$ with

   $\theta_e(g, x) = \theta_e(h, x)$ implies $g = h$.

Definition 3-2-10:

Suppose $(\mathbb{X}, \tilde{\beta}'), (\mathbb{Y}, \tilde{\beta}'')$ are two soft bornological sets. And let $(\mathbb{X}, \tilde{\beta}')$, $(\mathbb{Y}, \tilde{\beta}'')$ be $\mathbb{G}$- soft bornological sets with the soft (bounded) action θ_e and θ'_e. Then the map $\varphi_e: \mathbb{X} \to \mathbb{Y}$, is defined by $\varphi_e\big(\theta_e(g, x)\big) = \theta'_e\big(g, \varphi_e(x)\big)$, which is said to be $\mathbb{G}$- ***equivariant soft bounded map***.

That means

$$\varphi_e(g.x) = g.\varphi_e.$$

Clearly, φ_e is equivariant iff the diagram below is commutative

$$\begin{tikzcd}
\mathbb{G} \times \mathbb{X} \arrow[r, "\theta_e"] \arrow[d, "\rho_e"'] & \mathbb{X} \arrow[d, "\varphi_e"] \\
\mathbb{G} \times \mathbb{Y} \arrow[r, "\theta'_e"'] & \mathbb{Y}
\end{tikzcd}$$

where $\rho_e = id_{\mathbb{G}} \times \varphi_e$ is the product of identity mapping $id_{\mathbb{G}}$ of $\mathbb{G}$ and the mapping φ_e.

An equivariant map $\varphi_e: \mathbb{X} \to \mathbb{Y}$ which is also an isomorphism of soft bornological sets which is called an equivalence of $\mathbb{G}$- soft bornological sets.

In this case, we note that the inverse φ_e^{-1} of φ_e is also equivariant, if

$y = \varphi_e(x)$. Then

$$\varphi_e^{-1}(g, y) = \varphi_e^{-1}\big(g.\varphi_e(x)\big) = \varphi_e^{-1}\varphi_e(g.x) = g.x = g.\varphi_e^{-1}(y).$$

Definition 3-2-11:

If $(\mathbb{X}, \tilde{\beta}')$ be a $\mathbb{G}$- soft bornological set, and for each $x \in \mathbb{X}$. Then the ***soft orbit of x*** under the action $\mathbb{G}$ is the subset $\mathbb{G}(x) = \theta_e(\mathbb{G} \times x)$. i.e.

$$\mathbb{G}(x) = \{y \in \mathbb{X} : \exists\, g \in \mathbb{G}\ s.t, y = g.x\}.$$

By a generalization of this, for any soft bounded set $\tilde{B} \tilde{\subseteq} \mathbb{X}$, the union of all soft orbits of points of $\tilde{B}$ is

$$\mathbb{G}(\tilde{B}) = \{g \cdot b : g \in \mathbb{G}, b \in \tilde{B}\}.$$

For a soft bounded set $\tilde{B} \tilde{\subseteq} \mathbb{X}$, and soft subgroup $\mathbb{H}$ of $\mathbb{G}$, we put

$$\mathbb{H}(\tilde{B}) = \{h \cdot b : h \in \mathbb{H}, b \in \tilde{B}\}.$$

Remark 3-2-12:

1. The action θ_e of $\mathbb{G}$ on $\mathbb{X}$ defines an equivalence relation as follows.

For each $x, y \in \mathbb{X}, xRy$ if and only if there exist $g \in \mathbb{G}$ such that

$$\theta_e(g, x) = g \cdot x = y.$$

The equivalence classes with respect to this equivalence relation are the soft orbits of the elements of $\mathbb{X}$.

2. Let $\mathbb{X}/R$ denotes the set of soft orbits $\mathbb{G}(x)$, and $\pi_e : \mathbb{X} \to \mathbb{X}/R$ be the natural map taking x into its soft orbit $\mathbb{G}(x)$.

If we soft bornologize $\mathbb{X}/R$ by the quotient soft bornology, then the soft bornological set $\mathbb{X}/R$ is said the soft orbital soft bornological set of $\mathbb{X}$

(with respect to $\mathbb{G}$), or the quotient soft bornological set by $\mathbb{G}$ and is denoted by

$$\mathbb{X}/R = \{\mathbb{G}(x) : x \in \mathbb{X}\}.$$

3. Equivalently, soft bornological group $\mathbb{G}$ acts on a soft bornological set $\mathbb{X}$ and that $\mathbb{Y} = \mathbb{X}/\mathbb{G}$ is the corresponding orbit set. Let $\mathbb{Y}$ carry the quotient soft bornological generated by the orbital projection

$$\pi_e : \mathbb{X} \to \mathbb{X}/\mathbb{G}.$$

That means, a set $\tilde{B}_1 \tilde{\subseteq} \mathbb{Y}$ is soft bounded set in $\mathbb{Y}$ if and only if it is the image of soft bounded set $\tilde{B}$ in $\mathbb{X}$, such that $\tilde{B}_1 = \pi_e(\tilde{B})$. The soft bornological set $\mathbb{X}/\mathbb{G}$ so, obtained is called the soft orbital soft bornological set or the soft orbit space of the $\mathbb{G}$- soft bornological set $\mathbb{X}$.

3.3 Conclusions

In this work, we recalled the basic concepts of bornological structures that solve the problems of boundedness for sets and groups. Also, the fundamental construction for this structure such as bornological subset, product bornology. Furthermore, some practical applications of bornological space.

Consequently, we combined the soft set theory with bornology to construct a new structure that is called the soft bornological set to solve the problems of boundedness for soft sets. Also, we constructed a soft base and a soft subbase for this structure. Additional, we generated a new structure that elements are soft unbounded sets. And, we solved the problem of boundedness for a soft group by constructing a new structure called a soft bornological group such that the product map and the inverse map are soft bounded maps. The main important results, we proved that a family of soft bornological sets can be partial ordered sets, every soft power set of a soft set is soft bornological set, the composition of two soft bounded maps is a soft bounded map, the intersection of soft bornological sets is a soft bornological set, the left-right translation is a soft bornological isomorphism, and the product of soft bornological groups is soft bornological group.

Finally, to divide the soft bornological set into soft orbital classes, we studied a soft bornological group action SBGA. When a soft bornological group acts on a soft bornological set, this process is called soft bounded action. The main important result, we proved that a soft bornological group action is soft bornological isomorphism.

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The Publications

1. Ashwaq F. Abdal, Anwar N. Imran, Alaa A. Najm aldin, Amal O. Elewi, “ Soft Bornological Group Acts on Soft Bornological Set”, *Iraqi Journal of Science*, (Accept).

2. Ashwaq F. Abdal, Anwar N. Imran, Lieth A. Majed, “Soft Bornological structures”, *AIP Conference Proceedings*, (Accept).

المستخلص

الفضاء البرنلوجي هو بنية لحل مشاكل التقييد بالنسبة للمجموعات والزمر، والدوال بصورة عامة. الهدف الرئيسي من هذا العمل هو الدمج بين نظرية المجموعات اللينة والفضاء البرنلوجي لبناء هيكل جديد يسمى بنية برنلوجية لينة لحل مشاكل التقييد بالنسبة للمجموعات اللينة والزمر اللينة. أيضًا ، نقوم ببناء الاساس اللين والاساس الجزئي اللين لهذا الهيكل. من الطبيعي دراسة البنية الأساسية لهذا الهيكل الجديد مثل البرنلوجي الجزئي اللين ، وضرب البرنلوجي اللين. بالإضافة إلى ذلك، يتم إنشاء بنية جديدة تكون العناصر عبارة عن مجموعات لينة غير مقيدة. أخيرًا، ندرس إجراءً مقيد لين يعني أنه عندما تعمل زمرة برنلوجية لينة على مجموعة برنلوجية لينة، فإن هذه العملية تسمى فعل جماعي لين أو حركة ذات تقييد لين بحيث يكون تأثير الإجراء اللين هو تقسيم مجموعة البرنلوجي الى فئات مدارية لينة. النتائج الرئيسية المهمة: سوف نثبت أن عائلة من المجموعات البرنلوجية اللينة يمكن أن تكون مجموعة مرتبة جزئيا بواسطة علاقة ترتيب جزئي، تركيب دالتين مقيدة ولينة عبارة عن دالة مقيدة لينة، تقاطع مجموعات برنلوجية لينة هو مجموعة برنلوجية لينة، النقل من جهة اليسار أو جهة اليمين هو تماثل برنلوجي لين، ضرب الزمر البرنلوجية اللينة هي أيضًا زمرة برنلوجية لينة واجراء الزمرة البرنلوجية اللينة هو تماثل برنلوجي لين.

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كلية العلوم قسم الرياضيات

هياكل برنلوجية لينة

رسالة

مقدمة الى مجلس كلية العلوم في جامعة ديالى وهي جزء من متطلبات نيل درجة الماجستير في علوم الرياضيات

من قبل

أشواق فوزي عبدال

المشرف

أ.م .د. أنوار نور الدين عمران

أ .د. ليث عبداللطيف مجيد

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